Ph 77 - Advanced Physics Laboratory

Department of Physics, California Institute of Technology

- Electronics Track -

Transmission Lines

This next two-week period consists of two independent one-week laboratory assignments – Precision Measurements and Transmission Lines. To make sure that equipment is available to all when needed, please do the labs in the order assigned by your Track. Then combine both labs into a single e-notebook to be submitted according to the schedule on Canvas.

Introduction

In the Ph77 lab, we usually work with low-frequency signals transmitted over short distances, where cable transmission is not an issue. In this regime, it is reasonable to assume that the voltage at one end of a wire is equal to the voltage at the other end, and not give the matter any additional thought. But this approximation becomes inaccurate at high signal frequencies or long propagation distances.

Electronic signals typically propagate at nearly the speed of light, even though electron migration velocities in metals are orders of magnitude slower. The speed reflects the fact that electronic signals (and electrical power) in cables are mostly carried by the electric and magnetic fields, and not by the electron currents. At low frequencies, we rely on a shorthand construct of electricity flowing through wires like water flows through pipes. But working with transmission lines requires that we follow the electromagnetic fields.

Figure 1 shows a rough sketch of the electromagnetic fields inside a standard coaxial cable transmitting a high-frequency sinusoidal signal. The inner and outer conductors form an electromagnetic *waveguide*, and the propagating wave is defined by the time-varying electric and magnetic fields inside the waveguide. At a fixed position along the cable, the wave energy oscillates back and forth between the electric and magnetic fields, the process being described by a relevant wave equation. As illustrated in Figure 1, the electric fields arise from charges on the inner and outer

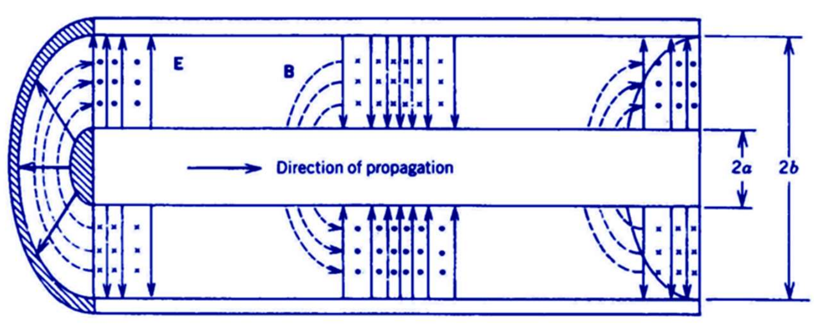


Figure 1. When a sinusoidal signal propagates down a coaxial cable, the energy oscillates back and forth between the electric and magnetic fields that are mostly confined inside the cable. This situation is not unlike that of a wave propagating down a string, where the energy oscillates between elastic and kinetic energy. In both cases, there is a corresponding wave equation that describes the wave propagation. Note that this sketch is grossly oversimplified and inaccurate; the actual EM fields are much more complex.

conductors, much like in a parallel-plate capacitor. The localized currents define what is essentially a toroidal inductor, generating magnetic fields confined inside the dielectric material between the conductors. The actual fields are much more complex than shown in this sketch, and you can find videos showing coaxial EM wave propagation online.

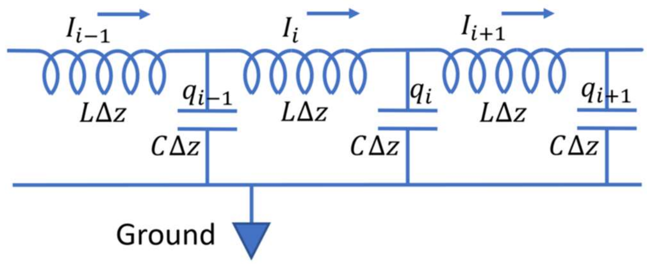


Figure 2. A lossless coaxial cable can be modeled as a series of capacitors and inductors, with Ground representing the outer conductor of the cable. Here L and C are the inductance per unit length and the capacitance per unit length, respectively.

An ideal (lossless) coaxial cable

An approximate wave equation for a lossless coaxial waveguide can be derived by looking at the time- and space-dependent behavior of the electromagnetic fields. And we can derive the relevant equations by defining the coaxial cable as a series of inductors and capacitors, as shown in Figure 2. Referring to this diagram, the voltage between the conductors is

$$V(z,t) = \frac{q(z,t)}{C} \tag{1}$$

where q is the charge per unit length on the conductors and C is the cable capacitance per unit length (assumed to be independent of z and t).

Differentiating this expression gives

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} \tag{2}$$

and the sign comes from the direction of the current shown in Figure 2. Similarly, the voltage drop along the cable comes from the cable inductance, giving

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \tag{3}$$

where L is the inductance per unit length, again assumed to be independent of z and t.

Putting these equations together yields the wave equation

$$\frac{\partial^2 V}{\partial t^2} = \frac{1}{LC}\frac{\partial^2 V}{\partial z^2} \tag{4}$$

and this equation supports waves propagating at a velocity

$$v_{prop} = \frac{1}{\sqrt{LC}} \tag{5}$$

One can calculate C and L by assuming a fixed charge (or current) inside the cable and calculating the electric (or magnetic) field energies that result. Doing so gives

$$C = \frac{2\pi\epsilon}{\ln{(b/a)}} \text{ (Farads/m)} \tag{6}$$

and

$$L = \frac{\mu}{2\pi}\ln(b/a) \text{ (Henrys/m)} \tag{7}$$

where a and b are the inner and outer radii of the cable conductors (see Figure 1). From this we obtain the propagation velocity

$$v_{prop} = \frac{1}{\sqrt{\mu\epsilon}} \tag{8}$$

If there were empty space between the coaxial conductors, then $\mu = \mu_0, \epsilon = \epsilon_0$, giving $v_{prop} = c$. For the BNC cable you will be using in this lab, $v_{prop} \approx 0.8c$ and the cable capacitance is $C \approx 80$ pF/meter.

The general solution of this wave equation yields forward and backward propagating waves that can be written as

$$V(z,t) = F_+\left(v - \frac{z}{t}\right) + F_-\left(v - \frac{z}{t}\right) \tag{9}$$

where F_+ and F_- are arbitrary functions. The reader can show that this $V(z,t)$ yields the accompanying current

$$I(z,t) = \frac{V(z,t)}{Z_c} \tag{10}$$

where $Z_c = \sqrt{L/C}$ is called the *characteristic impedance* of the cable. (The easiest way to derive this is to guess a solution I=AV and then show that this satisfies all the propagation equations with an appropriate constant. Then the usual uniqueness theorems for linear differential equations tell us that there can be no other solutions.)

For a lossless cable, Z_c is a real quantity equal to a simple resistance, and our cable has the standard $Z_c \approx 50\ \Omega$. (There are other cable standards as well, but this one is especially common in experimental physics.) As you will see next, this quantity is important when considering signal reflections from the ends of the cable.

Cable reflections

Next consider the setup shown in Figure 3, where a signal generator sends a sinusoidal signal down a long transmission line. A load impedance Z_L "terminates" the cable at the receiving end, and we model the signal generator as an ideal signal source connected through an output impedance Z_0. In the lab, the Rigol signal generator has $Z_0 = 50\ \Omega$.

Looking at the load end of the cable, consider sending a sinusoidal signal toward the load and see how it reflects off that end of the cable. Near the end, we can write the signal as the sum of two sinusoidal waves

$$V(z,t) = V_1 e^{i(\omega t - kz)} + V_2 e^{i(\omega t + kz)} \tag{11}$$

which includes an incoming wave with amplitude V_1 and a reflected wave with amplitude V_2.

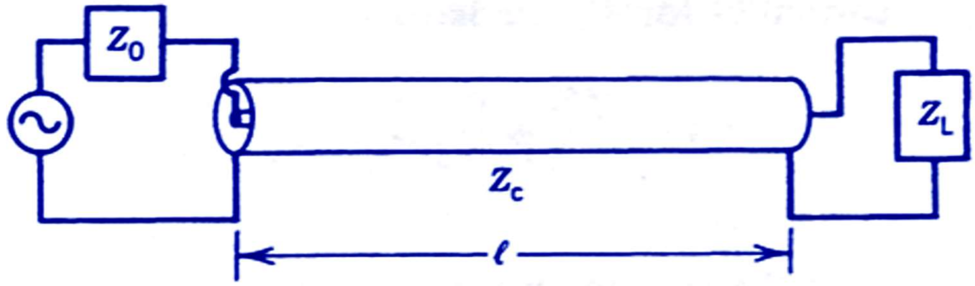


Figure 3, In this circuit a signal generator (left) sends a sinusoidal signal down a long coaxial cable. The cable is then "terminated" with a load impedance Z_L (right).

Exercise 1. Show that the reflection coefficient ρ is

$$\rho = \frac{V_2}{V_1} = \frac{Z_L - Z_c}{Z_L + Z_c} \tag{12}$$

and ρ is known as the reflection coefficient. Hint: The current in this situation is

$$I(z,t) = \frac{V_1 e^{i(\omega t - kz)} - V_2 e^{i(\omega t + kz)}}{Z_c} \tag{13}$$

and the sign flip here is a bit tricky, arising because of how the current direction was defined in Figure 2 relative to the z axis.

Add to this Ohm's law, which requires

$$Z_L = \frac{V(\ell,t)}{I(\ell,t)} \tag{14}$$

at the load end of the cable. Put all the pieces together to derive the reflection coefficient.

Exercise 2. Briefly describe the reflection of a signal pulse in the limiting cases where $Z_L = 0$ (where the end of the cable is shorted) and $Z_L = \infty$ (where the end of the cable is left open). Compare with the mechanical analog of waves traveling down a string, which you can view at: *https://www.youtube.com/watch?v=ZxIlyptT1FY*. In this video, fixing the end of the slinky (so the perpendicular displacement is zero) is analogous to the electrical case with $Z_L = 0$ (which holds the voltage at zero). Similarly, letting the end of the slinky move freely is analogous to the $Z_L = \infty$ case. The electrical and mechanical systems are similar here because the underlying wave equations (with corresponding boundary conditions) are comparable. For the $Z_L = \infty$ case, what is the magnitude of the pulse voltage at the end of the cable relative to the pulse voltage at other points along the cable?

A similar but somewhat more involved calculation reveals that reflections off the signal end of the cable gives

$$\rho = \frac{Z_0 - Z_c}{Z_0 + Z_c} \tag{15}$$

where Z_0 is the signal-generator output impedance shown in Figure 3.

Impedance matching

These calculations illustrate the concept of *impedance matching*. In the limiting cases of zero and infinite load impedance, the reflected signal has the same amplitude as the incoming signal, and the load does not absorb any of the input signal energy. This may seem odd when $Z_L = 0$, but no energy can be absorbed in an electrical circuit unless a non-zero resistance is present.

In the engineering world of signal transmission, reflected signals can be problematic, as unwanted reflected digital pulses may start appearing everywhere. The usual solution to this problem is to terminate all transmission lines with load impedances equal to the cable impedance. This yields no signal reflections and is called impedance matching.

This is why many oscilloscopes and other pieces of test equipment offer standard 50Ω input and output impedances. Sometimes it is nice to have a high input impedance to avoid loading the circuit being observed; this is typically the case at low frequencies, where signal reflections die out quickly compared to signal timescales. But at high frequencies (say above 10 MHz), signal reflections can make a mess of your signal, so in that regime you want to impedance match to get rid of signal reflections.

Laboratory Exercises

Figure 4 shows a schematic diagram of the circuit you will be using to examine transmission-line behavior in the lab, along with the layout of the actual printed circuit board. The circuit is designed with some flexibility in its operation, so please note the various circuit elements:

- Several BNC connectors (labeled BNC1-BNC7).

- Four potentiometers (also called "pots" or variable resistors, labeled P1-P4). For the P1/P2 pair, P1 is a "course adjust" and P2 is a "fine adjust". The same goes for the P3/P4 pair.

- A variety of "jumpers" (J1-J5) for selecting different circuit functions. A small "shunt" can be used to short the jumper pins; if no shunt is present, then the jumper pins are not connected. (Please do not lose the shunts.)

To begin, connect the long red (coiled) BNC cable to BNC3 and BNC4, and this becomes the transmission line you will be examining. This is an ultra-low-loss RG-8X coaxial cable with the following specifications:

Length = 100 feet (nominally) = 30.48 meters
Characteristic impedance = 50 Ω
Propagation velocity = 0.78c = 2.34e8 m/s
[The spec sheet says 0.82c, but our measurements suggest that 0.78c is closer to reality.]
Capacitance = 24.8 pF/ft = 81.37 pF/meter
Signal attenuation = 0.3 dB/100ft @ 1 MHz
0.9 dB/100ft @ 10 MHz
3.1 dB/100ft @ 100 MHz
11.2 dB/100ft @ 1 GHz

Next use a pair of short red cables (or other 1-foot-long cables) to connect your oscilloscope to BNC2 (ch1) and BNC5 (ch2), so ch1 shows the signal entering the cable (from BNC1) and ch2 shows the signal at the far end of the transmission line. With such short cables, the reflections off the oscilloscope will not cause much trouble. Because the Keysight oscilloscope has a high (1 MΩ) input impedance, it does not significantly affect the signal being observed. (Specifically, 1 MΩ is much higher than all

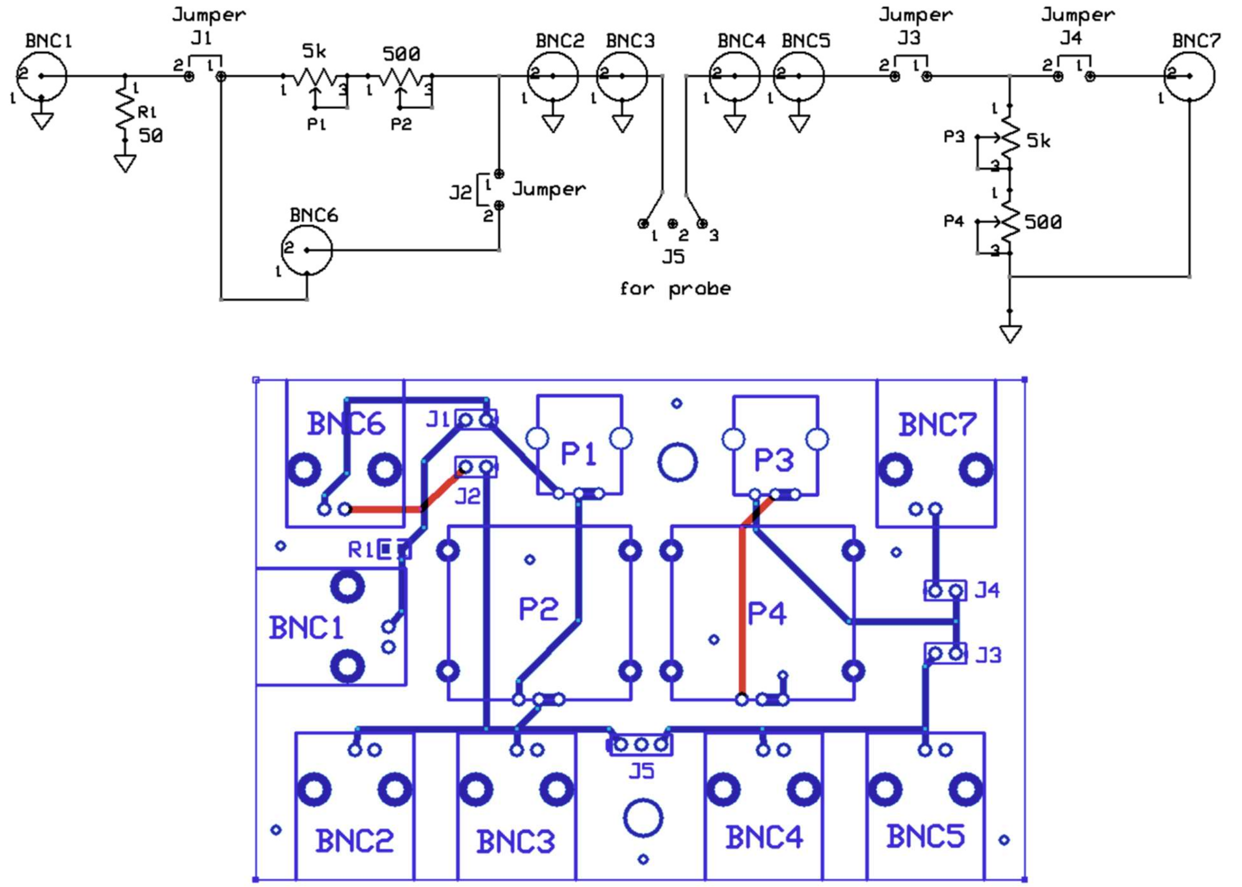
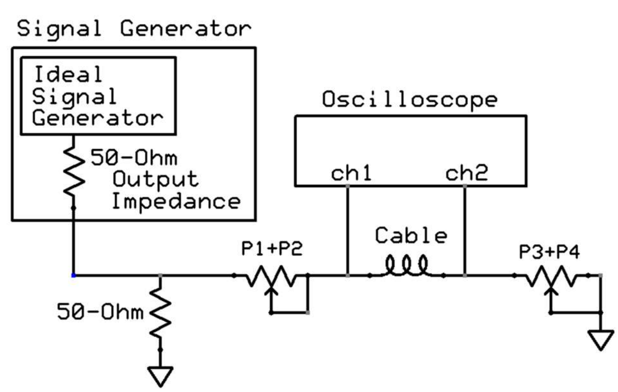


Figure 4. (Top) A schematic diagram of a test circuit used to look at transmission-line behavior in the lab. (Bottom) The printed-circuit board layout corresponding to the schematic diagram.

other impedances present: the cable impedance, the signal generator output impedance, and the potentiometer resistances.) Finally, place shunts on J1 and J3 while leaving the other jumpers open, and connect your high-speed signal generator (Siglent SDG 2042X, which generates 1.2 Gsamples/sec) to BNC1.

At this point it is useful to sketch a simplified "equivalent circuit" like that on the right, which shows only the essential elements of the full circuit in Figure 4. Note that the signal generator can be thought of as an "ideal" signal generator (with zero output impedance, capable of supplying a desired voltage signal no matter what) together with a 50Ω output impedance. This output resistor is needed because no physical signal generator can supply infinite current.



Next configure a 200kHz pulse signal with Vmin=0, Vmax=1V, and a pulse width of 100nsec. Set up external triggering on the 'scope, set P1+P2 to minimum R (both knobs fully CCW), and set P3+P4 to maximum R (both knobs fully CW). Set up external triggering on the 'scope and you should be able to reproduce the signal shown in Figure

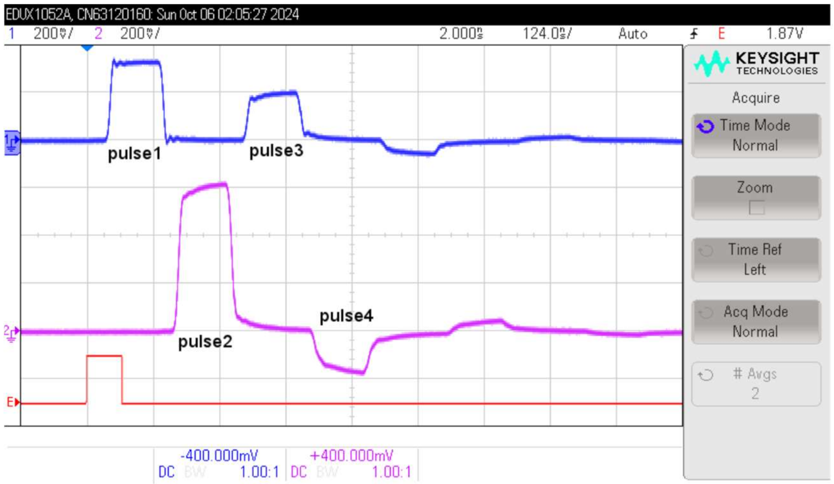


Figure 5. A pulsed signal with P1+P2=0 Ohms and P3+P4=5500 Ohms. The top trace (ch1) shows the input pulse and subsequent reflections at the front end of the cable. The bottom trace shows the first transmitted pulse and subsequent reflections at the back end of the cable. The bottom trace shows the external-trigger pulse from the signal generator.

5, with a pulse1 height close to 0.33V. [If you cannot reproduce this screen, make sure that P1 and P2 are both at their full minimum value, and try removing and replacing the shunts to get better connections.]

Exercise 3. Add your screenshot to your e-notebook and explain some features seen in the data:
a) What is the measured height of pulse1 in Figure 5 (use the 'scope cursors), and describe why you expect this value from transmission-line theory?
b) Why does pulse2 not have a flat-top structure? [Hint: increase the pulse width to 250nsec to observe how the pulse flattens out with time, and note how the signal attenuation (in the cable specs) changes with frequency. Does a frequency-dependent attenuation model make sense quantitatively?]
c) What is the measured height of pulse2 in Figure 5, and why do you expect this value? [For best results, measure the maximum pulse height after the pulse has flattened out.]
d) What is the measured height of pulse3, and why do you expect this value?
e) What is the measured height of pulse4, and why do you expect this value? Include the sign of the pulse height in your analysis.

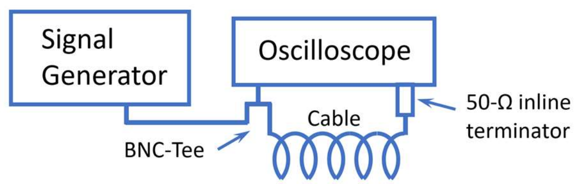
Exercise 4. Increase the value of P2 by a small amount, so that pulse1 and pulse3 have equal amplitudes and pulse2 goes to zero. [You will see some small "glitches" in the pulse2 edges and elsewhere. These come about because of the non-zero cable lengths going to the 'scope. If you try replacing the 1-foot-long ch1 cable with a longer one, for example, you will see these glitches get much worse.] Once again, measure the pulse heights and explain what is going on. Why are there no longer any pulses after pulse3? What does theory predict for the optimal value of P1+P2 in this configuration? Measure your P1+P2 value (using an ohmmeter on BNC6 and appropriate jumper settings). [Be careful not to change the P1+P2 knobs during the measurement process.] Set P1+P2 to the "correct" value and save a screenshot for your e-notebook.

Exercise 5. With P1+P2 set to give a good impedance match (no pulse reflections off the front end of the cable), set P3+P4 to 0Ω and analyze the results. Plug a 0Ω BNC shunt into BNC5 on the PCB (this gives a better short-circuit than P3+P4) and measure the pulse attenuation after a round-trip through the cable. Use a 200nsec pulse width to get a good measurement, then compare with expectations from the BNC cable specifications. (The difference likely comes from additional losses in the various connectors.)

Exercise 6. Measure the length of the cable (in meters) using the pulse travel time, assuming the signal propagation speed given in the cable specifications. [Pro tip: measure L first using a single pass through the cable; this is not super accurate, but you are less likely to make a mistake. Then measure L again using N travel times, thus increasing the accuracy by almost a factor of N.] Document your work in your e-notebook and note which cable (#1 or #2) you are using.

Exercise 7. Disconnect BNC4 (so the back end of the cable is disconnected) and set P1+P2 to its maximum value (nominally equal to 5500Ω). Potentiometers tend to have large uncertainties, so measure P1+P2 at the maximum setting for your PCB. Send a 1kHz square wave into BNC1 and use the observed ch1 signal to measure the cable capacitance. (Sketch an equivalent circuit where the cable is a capacitor.) Then use the cable specs to produce another measurement of the cable length. Document this exercise, then check your capacitance measurement using the Keysight 34461A (as a capacitance meter).

Exercise 8. As a final exercise in this series, set up the circuit shown on the right and go back to the 200 kHz pulse signal you used previously. This is what pulse transmission is supposed to look like when everything is done correctly. A pulse coming out of the signal generator is fully absorbed at the oscilloscope by the 50Ω inline terminator. Next remove the inline terminator and see what happens. Now the pulse is reflected at the oscilloscope, but it is fully absorbed by the 50Ω output impedance of the signal generator. Document both signals in your e-notebook.



Exercise 9. Set up the circuit shown on the right (using the same red RG-8X cable and configure the SA to 0-100MHz with RBW=300kHz. You should see a series of dips on the screen. Add a screenshot to your e-notebook and explain what is happening in this transmission-line system. Zoom in on the first signal minimum to measure the one-way travel time through the cable. Compare with your previous measurement.