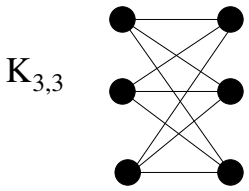
Introduction to Discrete Mathematics Ma/CS 6a Fall 2005-06 **Homework 8**

Due: Monday November 21st, 1pm.

1) Show that $K_{3,3}$ (see picture) is not planar.



2) 12.3.6

3) 13.4.5

4) Let G be a planar graph in which all vertices have degree at least 5. Is it true that G must have two vertices of degree 5 that are joined by a path of length less than 100? Is it true that if $|V(G)| > 10^6$ then G has at least 13 vertices of degree 5?

5) Let G be a planar graph in which all vertices have degree at least 5 and suppose there is no edge which can be deleted to obtain a disconnected graph (i.e. every edge belongs to two faces). For each vertex v let $S(v) = \frac{1}{2} - \frac{1}{d(v)}$. Show that for some face f we have $\sum_v S(v) < 1$, where the sum is over all vertices v on the boundary of f. Deduce that G has an edge xy with $d(x) = 5$ and $d(y) \leq 6$.

6) 13.4.9