Introduction to Discrete Mathematics Ma/CS 6a Fall 2005-06 **Homework 7**

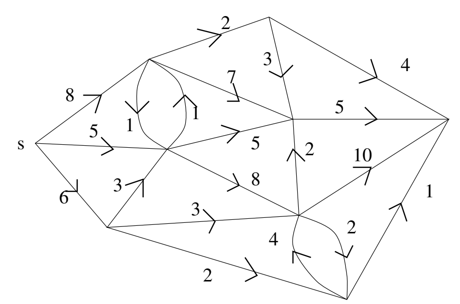
Due: Monday November 14th, 1pm.

1) 10.4.9

2) Let G be a bipartite graph in which every vertex has degree at most k. Show that one can find k matchings that cover all of the edges in G.

3) A permutation matrix is a matrix where every entry is 0 or 1, each row has exactly one 1 and each column has exactly one 1. Suppose M is an n by n matrix with non-negative integer entries, every row of m has sum k and every column of m has sum k. Show that M is a sum of k permutation matrices.

4) Find a maximum value flow from s to t in the transportation network shown and a cut demonstrating that your flow has maximum value.



5) Let G be a transportation network with source s and sink t. Let f be a flow with value v and suppose there is some flow with value larger than v. Must there be a flow f' with value larger than v so that $f'(e) \geq f(e)$ for every edge e?

6) Let G be a graph and suppose s and t are different vertices of G. Let k be the largest number so that there are paths $P_1, \cdots, P_k$ so that each P_i starts at s and ends at t and no two paths P_i, P_j share an edge. Let ℓ be the smallest number so that it is possible to delete ℓ edges from G to obtain a graph in which there is no path from s to t. Show that $k = \ell$. (Hint: use the Max Flow - Min Cut Theorem.)