Number Theory for Beginners Ma3 2007 **Homework 3**

Instructions: All problems can be solved by hand, and computer verification will not be considered a complete answer.

Due: Thursday May 17, 5pm.

1. What are the last 2 digits of $123456789^{123456789}$?

2. Does $x^2 \equiv 621 \bmod 1111$ have a solution?

3. Find a closed form for the number with continued fraction expansion $1+\frac{1}{2+}\frac{1}{3+}\frac{1}{1+}\frac{1}{2+}\frac{1}{3+}\cdots$ (infinitely repeating).

4. Consider the two-dimensional standard lattice, meaning all points (x, y) with both co-ordinates being integers. Show that different points have different distances from the point $(\sqrt{2}, 1/3)$.

5. Show that there are no solutions of $x^2 + y^2 + z^2 = 2xyz$ in integers apart from $x = y = z = 0$.

6. Using the formula $e = \sum_{i=0}^{\infty} 1/i!$ show that the base of natural logarithms is irrational.

7. Find all pairs of consecutive numbers $(n, n+1)$ in which one number is a power of 2 and the other is a power of 3.