

Due: Thursday November 16, 5pm.

1. Show that $R(3, 4) = 9$ and $R(4, 4) = 18$.

(See page 29 and hints to problems 3C and 3D in the textbook.)

2. Show that for any r there is n so that for any $C : [n] \rightarrow [r]$ there are $x, y, z \in [n]$ with $C(x) = C(y) = C(z)$ and $x + y = z$.

3. Suppose d and r are natural numbers, S is a finite subset of \mathbb{Z}^d and $C : \mathbb{Z}^d \rightarrow [r]$ is a colouring. Show that there are $0 \neq a \in \mathbb{Z}$ and $v \in \mathbb{Z}^d$ so that C is constant on $aS + v = \{as + v : s \in S\}$.

4. The chromatic number $\chi(G)$ of a graph G is the smallest number t such there is a ‘colouring’ function $f : V(G) \rightarrow [t]$ such that adjacent vertices get different values.

For graphs G_1, G_2 the graph Ramsey number $R(G_1, G_2)$ is the smallest number n so that if the edges of K_n are all coloured red or blue then there must either be a copy of G_1 with all edges red or a copy of G_2 with all edges blue.

(i) Show that any graph with chromatic number t contains a subgraph with chromatic number t and minimum degree at least $t - 1$.

(ii) Let T be a fixed tree with t vertices. Show that any graph with chromatic number t contains a subgraph isomorphic to T .

(iii) Let T be a fixed tree with t vertices. Show that $R(K_s, T) = (s - 1)(t - 1) + 1$.