Combinatorial Analysis Ma121a 2006 **Homework 6**

Due: Thursday November 9, 5pm.

1. Let $a_1, \cdots, a_n$ be real numbers with $|a_i| \geq 1$ for all i and let r be another real number. Show that there are at most $\binom{n}{\lfloor n/2 \rfloor}$ sets $I \subset [n]$ for which $r < \sum_{i \in I} a_i < r + 1$.

2. Given a set $T \subset [n]$ write $[n]^{(T)} = \cup_{i \in T} [n]^{(i)} = \{X \subset [n] : |X| \in T\}$. Let $f(n,t) = \max_{|T|=t} |[n]^{(T)}|$. Suppose that $\mathcal{A} \subset \mathcal{P}[n]$ contains no chain of length $t+1$. Show that $|\mathcal{A}| \leq f(n,t)$.

3. Suppose $\mathcal{A} \subset [n]^{(r)}$ with $|\mathcal{A}| \geq r+1$ and that any $r+1$ sets of $\mathcal{A}$ have a common element. Show that all the sets in $\mathcal{A}$ have a common element.

4. Show that for any t, r there is n_0 so that if $n \geq n_0$ and $\mathcal{A} \subset [n]^{(r)}$ and every pair of sets from $\mathcal{A}$ have at least t common elements then $|\mathcal{A}| \leq \binom{n-t}{r-t}$.

Hint: Consider cases according to whether there is a set T with $|T| = t$ with $T \subset A$ for all $A \in \mathcal{A}$. If there is such a T the bound is easy. If there is not show how to find $A_1, A_2, A_3 \in \mathcal{A}$ with $|A_1 \cap A_2 \cap A_3| < t$ and use these sets to get an upper bound on $|\mathcal{A}|$.