**Combinatorial Analysis Ma121a 2006** **Homework 3**

Due: Thursday October 19, 5pm.

1. Let $x_1, \cdots, x_n$ be vectors of norm at least 1 in a Euclidean space. Show that at most $n^2/4$ of the vectors $\{x_i + x_j : i \neq j\}$ have norm less than 1.

2. Show that a graph with $n$ vertices and $e$ edges contains at least $\frac{e}{3n}(4e - n^2)$ triangles. [1]

3. For each of the following graphs $H$ determine the maximum number of edges in an $H$-free graph $G$ on $n$ vertices.

(i) $H$ consists of two triangles that share a common edge

(ii) $H$ consists of two triangles that share a common vertex (but not an edge)

(iii) $H$ consists of two disjoint triangles

4. An $r$-graph $G$ has a vertex set $V(G)$ and an edge set $E(G)$, in which each edge is a subset of $V(G)$ of size $r$. (So a 2-graph is a normal graph.) Show that for any integers $r, t > 0$ there is a real $c > 0$ so that if $G$ is an $r$-graph on $n$ vertices with $|E(G)| \geq n^{r-c}$ we can find disjoint subsets $V_i \subset V$, $|V_i| = t$ for $1 \leq i \leq r$ so that for any choices of points $v_i \in V_i$, $1 \leq i \leq r$ we have $\{v_1, \cdots, v_r\} \in E(G)$. [2]

[1] This is problem 4C in the book. There is a hint in the back.

[2] Hint: For the case $r = 2$ we proved something stronger than this in class. The same idea can be used to give an induction on $r$.