

Due: Thursday October 12, 5pm.

1. Let D be a directed graph. Show that $V(D)$ can be partitioned into sets V_1, \dots, V_t , for some t , with the following two properties:
 - (i) The restriction of D to each V_i is strongly connected.
 - (ii) If (the ordered pair) (a, b) is an edge of D , with $a \in V_i$ and $b \in V_j$, then $i \leq j$.

2. A tournament on n vertices is an orientation of the complete graph K_n .¹ Show that any tournament has a directed path that passes through all the vertices.

3. Show that any strongly connected tournament has a directed cycle that passes through all the vertices.

4. Show that K_n can be partitioned into paths of different lengths.

¹We can imagine that the vertices represent players in a round-robin competition, and that the direction of the edge specifies which player beats the other.