Combinatorial Analysis Ma121a 2004 **Homework 7**

Due: Tuesday November 16, noon.

1. Find a closed formula for each of the following expressions:

(i) $\sum_{k\in\mathbb{Z}}\binom{n}{2k}$,

(ii) $\sum_{k\in\mathbb{Z}}\binom{n}{3k}$, [1]

(iii) $\sum_{k\in\mathbb{Z}}\binom{u}{k}\binom{v}{m-k}$,

(iv) $\sum_{k\in\mathbb{Z}}\binom{p}{k}\binom{q}{k}\binom{k}{j}$,

(v) $\sum_{k\in\mathbb{Z}}(-1)^k\left\{{n \atop k}\right\}\left[{k \atop j}\right]$.

(vi) $\sum_{n\le x}\mu(n)\lfloor x/n\rfloor$. [2]

(Assume all variables apart from x are integers.)

2. Let A be the n by n matrix with $a_{ij}=\binom{i}{j}$ for $0\le i,j\le n-1$. Determine A^{-1}. [3]

3. Show that the number of maps $f:[r]\to[n]$ with $|\{j : f(j)\in[i]\}|<i$ for all $i\in[n]$ is $(n-r)n^{r-1}$ for $r\in[n]$.

4. What is the expected number of cycles in a random permutation in S_n?

Notation: $\left[{k \atop j}\right]$ is a Stirling number of the first kind, $\left\{{n \atop k}\right\}$ is a Stirling number of the second kind, $\mu(n)$ is the Möbius function

[1] Problem 13J

[2] Problem 10C

[3] Problem 13H