Combinatorial Analysis Ma121a 2004 **Homework 5**

Due: Monday November 1, noon.

1. Let G be a bipartite graph in which every vertex has degree at most k. Show that G is a subgraph of a bipartite graph in which every vertex has degree k. Deduce that G can be partitioned into k matchings.

2. (i) Say that vertices u, v in G are k-linked if there are k mutually edge-disjoint paths from u to v. Suppose u, v, w are distinct, u, v are k-linked and v, w are k-linked. Does it follow that u, w are k-linked?

(ii) Say subsets $X, Y \subset V(G)$ are k-joined if $|X| = |Y| = k$ and there are k mutually vertex-disjoint paths from X to Y. Suppose $X, Y, Z \subset V(G)$, X, Y are k-joined and Y, Z are k-joined. Does it follow that X, Z are k-joined?

3. Let G be a transportation network with source s and sink t. Let f be a flow with value v and suppose there is some flow with value larger than v. Must there be a flow f' with value larger than v so that $f'(e) \geq f(e)$ for every edge e?

4. Let T be a tree. Show that T has a perfect matching [1] if and only if for every vertex v exactly one component of $T \backslash v$ has an odd number of vertices.

5. Let G be a bipartite graph with parts X and Y. Suppose that for each $v \in V(G)$ there is a total order $<_v$ on its neighbourhood $N(v)$. We say that a matching M in G is stable if whenever we have $x \in X$, $y \in Y$ with $xy \notin M$ then either $xy' \in M$ for some $y' >_x y$ or $x'y \in M$ for some $x' >_y x$. Show that there is a stable matching.

[1] i.e. a matching covering all vertices