Combinatorial Analysis Ma121a 2004 **Homework 2**

Due: Monday October 11, noon.

1. Show that a graph is bipartite [1] if and only if it does not contain an odd cycle [2] .

2. A tournament on n vertices is an orientation of K_n, [3] i.e. a directed graph in which for every pair of vertices u, v there is either an edge from u to v or an edge from v to u (but not both). [4]

Show that any tournament has a directed path that passes through all the vertices.

3. We will usually assume that graphs are finite in this course, but here is a question on infinite graphs. We say that an infinite graph $G = (V, E)$ is one-way Eulerian if there is an infinite walk $v_1 v_2 v_3 \cdots$ that uses all the edges.

Let G be a connected infinite graph with countably many edges. Suppose that some vertex v_1 has odd degree, and that the degree of all other vertices is even or infinite. Show that G is one-way Eulerian if and only if for every finite set $E' \subset E$ the graph $G \backslash E'$ has only one infinite component.

4. Show that K_n can be partioned into paths of different lengths.

[1] A graph is called bipartite if there is a partition of its vertex set into two parts so that every edge has one end in each part.

[2] i.e. a cycle with an odd number of edges

[3] K_n denotes the complete graph on n vertices, i.e. every pair of vertices is adjacent, but we do not include any loops or parallel edges.

[4] We can imagine that the vertices represent players in a round-robin competition, and that the direction of the edge specifies which player beats the other.