

Due: Thursday December 9, noon.

1. Show that $K_{3,3}$ is not planar.

2. Let G be a planar graph with no cut-edge, having minimum degree 5.
 - (i) Is it true that G must have two vertices of degree 5 that are joined by a path of length less than 100? Is it true that if $|V(G)| > 10000$ then G has at least 13 vertices of degree 5?
 - (ii) For each vertex v let $S(v) = \frac{1}{2} - \frac{1}{d(v)}$. Show that for some face f we have $\sum_{v \sim f} S(v) < 1$, where the sum is over all vertices in the boundary of f . Deduce that G has an edge xy with $d(x) = 5$ and $d(y) \leq 6$.

3. Show that an Eulerian planar graph in which every face is a triangle is 3-colourable.

4. Show that a graph with no K_4 minor is 3-colourable.

5. Show that a graph with chromatic number larger than t has a $K_{2,t-1}$ minor.