Combinatorial Analysis Ma121a 2004 **Homework 1**

Due: Monday October 4, noon.

1. Prove or disprove each of the following statements.

(i) Every simple graph with at least two vertices has two vertices with the same degree.

(ii) Every connected graph has a vertex that can be deleted to leave a connected graph.

(iii) If u, v, w are vertices of a graph G such that there is a path of even length from u to v and a path of even length from v to w, then then there is a path of even length from u to w.

2. Let T be a tree and let $T_1, T_2, \ldots, T_k$ be subtrees of T, every two of which have a vertex in common. Show that some vertex of T belongs to all of the subtrees.

3. Let G be a graph with vertex set V. If U is a subset of V we write $G[U]$ for the restriction of G to U, i.e. the graph with vertex set U and edge set all those edges of G with both ends in U.

Show that there is a partition $V = V_1 \cup V_2$ for which $e(G[V_1]) + e(G[V_2]) \leq e(G)/2$.

Show that we can also require that $e(G[V_1]) \leq e(G)/3$ and $e(G[V_2]) \leq e(G)/3$.

Warning! The textbook uses a non-standard definition of the term 'path'. We are using the standard meaning: a path is a walk with no repeated vertices or edges. (The textbook calls this a 'simple path'.)