

Gamma frequency firing predominates in threshold behavior of the stochastic Hodgkin–Huxley equations.

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Here we study the distribution of inter-spike intervals (ISIs) generated by the stochastic Hodgkin-Huxley equations. It is generally assumed that the ISI distribution takes the form of a negative exponential (Poisson process). By an approximate analysis, Chow and White (1996) derived a negative exponential distribution for the ISIs generated by channel noise in the subthreshold Hodgkin-Huxley equations. As a result of extensive numerical simulations, we find that for most values of membrane area and injected current, there are far more short ISIs than predicted by this result. This is shown by a large initial peak in the ISI histograms (Figure 1). Moreover, when each ISI is converted to a frequency and the frequencies histogrammed, this peak translates to a peak in the frequency distribution in the gamma range, 40-100 Hz (Figure 2).

We used the standard Hodgkin-Huxley parameters, and methods described by Gillespie (1977) and Chow and White (1996), to simulate the stochasticity caused by large numbers of Na- and K-channels.

The underlying mechanism arises from the presence of a subcritical Hopf bifurcation involving a stable limit cycle (SLC), an associated unstable limit cycle (ULC), and a stable fixed point. In a region of phase space where the concentric stable and unstable limit cycles are very close, the noise sometimes perturbs the system state off the SLC into the interior of the ULC, and towards the fixed point. The system escapes from the fixed point by a random walk outwards and across the unstable limit cycle, thus enabling it to rejoin the SLC. This random walk gives rise to the exponential tail in the ISI histogram. If the system state is not perturbed off the SLC, another spike is emitted. This gives rise to the large initial Gaussian peak in the ISI histogram. We illustrate the mechanism in detail for the Morris-Lecar neuron model. For values of the injected current below the Hopf bifurcation value, where there is no SLC or ULC, a closely related explanation can be given, utilising the pseudo-trajectories that exist prior to a Hopf bifurcation.

Channel noise is not essential: any source of noise, for example synaptic noise, can play the same role. Since the underlying mechanism is present in any neural system near a sub-critical Hopf bifurcation, the effect should be widespread. Biological data very similar to figure 1 is found in olfactory receptor neurons in the frog (Rospars et al 1994).

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Chow, C. C. and J. A. White (1996). "Spontaneous action potentials due to channel fluctuations." Biophysical Journal 71: 3013-3021.

Gillespie, D. T. (1977). "Exact stochastic simulation of coupled chemical reactions." J. Physical Chemistry 81(25): 2340-2361.

Rospars, J.-P., P. Lansky, et al. (1994). "Spontaneous activity of first- and second-order neurons in the frog olfactory system." Brain Research 662: 31-44.

Figure 1. ISI histogram for membrane patch with 3 nA applied current

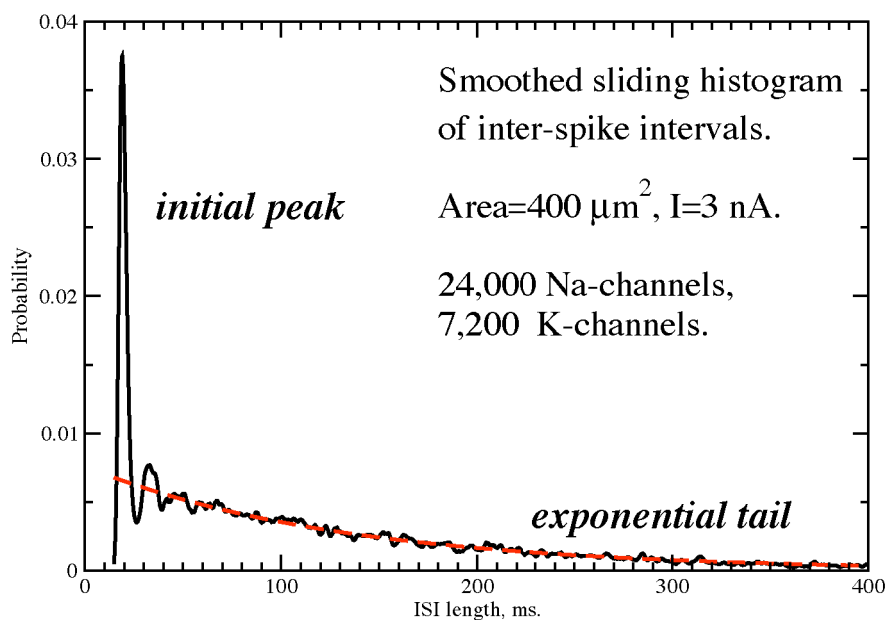


Figure 2. Corresponding frequency histograms for range of currents

