

# Physics 1c Practical, Spring 2015

## HW 1 Solutions

April 2, 2015

### 1 Serway 30.7 (5 points)

By superposition, it is trivial to see that the field at the center of the loop is equal to the field at the center due to the loop plus the field at the center of the loop due to the line. Note, by Biot-Savart:

$$\begin{aligned}\vec{B}_{\text{loop}} &= \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{|\vec{r}|^2} \\ &= \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{R d\theta}{R^2} (-\hat{z}) \\ &= \frac{\mu_0 I}{2R} (-\hat{z})\end{aligned}\tag{1}$$

where  $-\hat{z}$  is into the page.

Also:

$$\begin{aligned}\vec{B}_{\text{line}} &= \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \hat{r}}{|\vec{r}|^2} \\ &= \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{\frac{R}{\sqrt{R^2+x^2}} dx}{R^2+x^2} (-\hat{z}) \\ &= \frac{\mu_0 I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\cos\theta (R \sec^2\theta d\theta)}{R^2 + R^2 \tan^2\theta} (-\hat{z}) \\ &= \frac{\mu_0 I}{4\pi R} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta (-\hat{z}) \\ &= \frac{\mu_0 I}{2\pi R} (-\hat{z})\end{aligned}\tag{2}$$

Note it would have been simpler to use Amperes law to calculate the magnitude of  $\vec{B}$  due to the infinite line using an Amperian loop pierced by the line:

$$2\pi R |\vec{B}| = \mu_0 I\tag{3}$$

At any rate,

$$\vec{B}_{\text{total}} = \frac{\mu_0 I}{2R} \left(1 + \frac{1}{\pi}\right) (-\hat{z})\tag{4}$$

And using  $I = 1.00\text{A}$ ,  $R=15.0\text{cm}$ , we obtain:

$$|\vec{B}| = 5.52 \times 10^{-6} \text{ T}\tag{5}$$

with the direction into the page.

**2 Serway 30.49 (5 points)****(a) (2 points)**

If the magnetic moment of the earth is  $\vec{M}$ , and the magnetic moment of an unpaired electron is  $\vec{m}$ , then we must have  $\vec{M} = N\vec{m}$ . This implies that:

$$N = \frac{|\vec{M}|}{|\vec{m}|} = \frac{8.00 \times 10^{22} \text{ A m}^2}{9.27 \times 10^{-24} \text{ A m}^2/\text{electron}} = 8.63 \times 10^{45} \text{ electrons} \quad (6)$$

**(b) (3 points)**

Note:

$$\begin{aligned} M_{\text{Iron}} &= \text{Number of Iron Atoms} * \text{Mass of Iron Atom} \\ &= \frac{\text{Number of unpaired electrons}}{2} * \frac{\text{Mass density of iron atoms}}{\text{Number density of iron atoms}} \\ &= \left( \frac{8.63 \times 10^{45}}{2} \text{ iron atoms} \right) * \frac{7900 \text{ kg/m}^3}{8.50 \times 10^{28} \text{ iron atoms/m}^3} \\ &= 4.01 \times 10^{20} \text{ kg} \end{aligned} \quad (7)$$

### 3 Serway 30.50 (5 points)

Note, we are given that:

$$|\vec{B}| = \mu_0 |\vec{M}| \quad (8)$$

where  $M$  is the magnetic moment per volume. Note that:

$$\begin{aligned} |\vec{M}| &= \text{magnetic moment per atom} * \text{number of atoms per volume} \\ &= \text{magnetic moment per electron} * \text{electrons contributing per atom} * \text{number of atoms per volume} \\ &\equiv |\vec{m}| * N * \rho \end{aligned} \quad (9)$$

where we have introduced  $m$ ,  $N$ , and  $\rho$  to symbolize their respective descriptions in the line above. So:

$$\begin{aligned} N &= \frac{|\vec{B}|}{\mu_0 |\vec{m}| \rho} \\ &= \frac{2 \text{ T}}{\mu_0 (9.27 \times 10^{-24} \text{ A m}^2/\text{electron}) (8.50 \times 10^{28} \text{ atoms/m}^3)} \\ &= 2.02 \text{ electrons/atom} \approx 2 \text{ electrons/atom} \end{aligned} \quad (10)$$

**4 Serway 30.62 (5 points)****(a) (2 points)**

Note that the radius of the hoops is 100 times greater than their separation, so the magnetic field on a point on loop 2 due to loop 1 can be approximated as if loop 1 was an infinite straight line, i.e.:

$$B_{12} = \frac{\mu_0 I_1}{2\pi a} \quad (11)$$

where  $a$  is the separation  $a = 1.00$  mm. Then the force on the loop is:

$$\begin{aligned} F_{12} &= \frac{\mu_0 I_1 I_2}{2\pi a} 2\pi R \\ &= \mu_0 I_1 I_2 \frac{R}{a} \\ &= 2.46 \text{ N} \end{aligned} \quad (12)$$

The force is repulsive.

**(b) (1 point)**

Equation 30.7 gives the magnetic field on the axis of the loops. To find the force we need the magnetic field on the loop itself, so equation 30.7 does not help us per se.

**(c) (2 points)**

From  $\vec{F} = m\vec{a}$ , we see that:

$$\begin{aligned} \vec{F}_B + \vec{F}_g &= m\vec{a} \\ F_B - mg &= ma \\ a &= F_B/m - g \\ a &= 107.3 \text{ m/s}^2 \end{aligned} \quad (13)$$

## 5 QP 1 (5 points)

The energy of the electron after accelerating through 20000 V is obviously 20000 eV. The maximum possible force on the electron in the tube is if the magnetic field is perpendicular to the velocity of the particle, and that force will be  $F = evB$ , for  $B \approx 5 \times 10^{-5}$  T.

We know that:

$$\frac{mv^2}{R} = evB \quad (14)$$

where R is the radius of the motion of the particle due to earth's B field. So:

$$R = \frac{mv}{eB} \quad (15)$$

Since the particle moves in an circular path of radius R and moves a distance L horizontally, we can see that:

$$\begin{aligned} (R - d)^2 + L'^2 &= R^2 \\ L'^2 &= 2dR \\ d &= \frac{L'^2}{2R} \end{aligned} \quad (16)$$

So, putting it all together:

$$\begin{aligned} d &= \frac{L'^2}{2 \frac{mv}{eB}} \\ &= \frac{L'^2 eB}{2mv} \\ &= \frac{L'^2 eB}{2m \sqrt{2V/m}} \\ &= 1.31 \text{ cm} \end{aligned} \quad (17)$$

**6 QP 24 (5 Points)****(a) (3 points)**

The upward force on the system will be due to the horizontal wire in the magnetic field. The force on the system due to the magnetic field is  $F = ILB$  upwards. So we want to determine the value of  $I$  when  $I_0LB_0 = Mg$ . So:

$$I_0 = \frac{Mg}{LB_0} \quad (18)$$

and:

$$V = \frac{MgR}{LB_0} \quad (19)$$

The current must be going counterclockwise in order to produce an upwards force.

**(b) (2 points)**

We now have  $F_B = I_0L(B + \Delta B) = mg + I_0L\Delta B$ , so the net force is  $F_{\text{net}} = I_0L\Delta B$ , so the acceleration will be:

$$\vec{a} = \frac{I_0L\Delta B}{M}\hat{z} \quad (20)$$

where  $\hat{z}$  is upwards.