Physics 106c **Final Examination** **06/05/12**

This examination is open book. You may consult your class notes, your own graded problem sets, and the solutions to the homework posted on the web. No other written material may be used. No computer algebra or similar programs may be used.

You have **4 hours** in which to work, by yourself, on this exam. You must do the exam in one sitting, although two short (~10 minute) breaks are OK.

Do not consult with anyone about this exam until after 5:00 pm, Friday, June 11.

Be sure to sign your name on each page of your exam.

EXAM DUE DATE/TIME:

Seniors and graduate students: 5:00 pm, Friday, June 8.

Undergraduates: 5:00 pm, Friday, June 15.

Please turn it in to Loly Ekmekjian in 151 Bridge Annex.

As always, you are bound by the Honor Code in all matters concerning this exam.

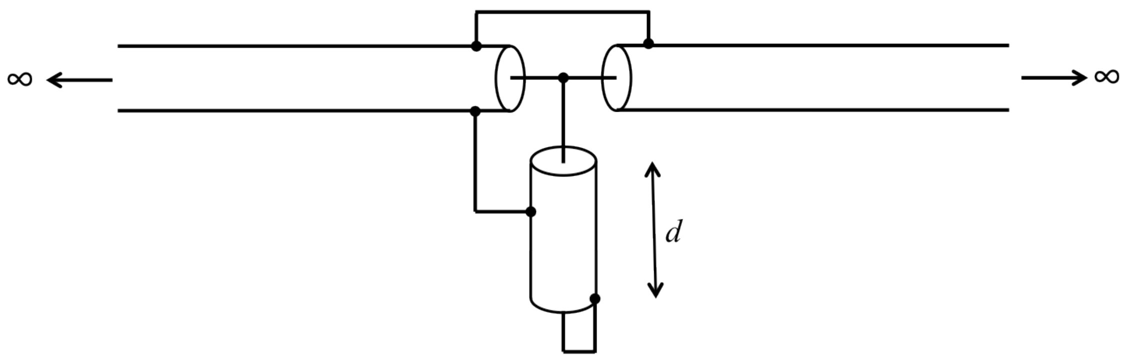
Do not proceed to the next page until you are ready for the 4 hour clock to begin.

Good luck, and have a great summer!

Problem 1 **(20 points)**

a) (5 points) A sinusoidal voltage wave $V(x,t) = V_0 e^{i(\omega t - kx)}$ is propagating down a coax with characteristic impedance R_c. What is the time-average energy flux associated with this wave?

Consider the following "T-junction" of three coaxes:



The coaxes on the left and on the right are infinitely long, while the middle coax has a finite length d. As the diagram suggests, the inner conductor of all three coaxes are joined together at the T-junction; you may consider this junction as a single point. The outer conductors are also all connected together. Finally, the middle coax has a short circuit termination at its opposite end. All of the various connections and conductors are perfect; they present no ohmic resistance. The coaxes all have the same characteristic impedances R_c and propagation velocities v.

A sinusoidal voltage wave is incident upon the T-junction from the left.

b) (10 points) Find the amplitude of the voltage wave which is transmitted off to infinity on the right and the voltage wave which is reflected back to the left.

c) (5 points) Under what conditions does the T-junction transmit all of the energy in the incident wave and reflect none? For these conditions, how is d related to the wavelength λ?

Problem 2 **(10 points)**

As I mentioned in class, a classical hydrogen atom is not stable. Make an *estimate* of the lifetime, in seconds, of such an atom. For this estimate you may start by assuming that the electron is orbiting a distance $r \sim 5 \times 10^{-11}$ m from the proton. I stress that only an estimate is required; no heavy calculations are needed! (For reference, the mass of an electron is $m_e \sim 10^{-30}$ kg.)

Problem 3 **(15 points)**

A linearly polarized plane electromagnetic wave of frequency ω, propagating in vacuum along the *z*–axis with an electric field of amplitude E_0 polarized along the *x*–axis, is normally incident upon a very thin metal sheet. The metal sheet lies in the x-y plane.

This particular metal is peculiar. Instead of the usual ohmic conductivity relation, $\boldsymbol{J} = \sigma \boldsymbol{E}$, this metal behaves as follows:

$$K_x = \sigma_H E_y$$
$$K_y = -\sigma_H E_x$$

In other words, the *sheet current density* $\boldsymbol{K}$ in the thin metal plate is related to the electric field $\boldsymbol{E}$ via a new conductivity coefficient, σ_H. Note that the *x*-component of the current density is determined by the *y*-component of the electric field, and the *y*-component of the current density by the *x*-component of the electric field. Note also the minus sign. While this situation may seem strange to you, it is in fact readily encountered when certain metals are subjected to intense static magnetic fields. Do not worry about how this situation comes about; just assume it is true.

a) (10 points) Find the reflected and transmitted electromagnetic waves, being careful to specify their polarization states completely.

b) (5 points) What fraction of the incoming power is absorbed? Justify your answer.

Problem 4 **(15 points)**

In class we discussed how the conductivity σ of conductors changes at very high frequency. We found that $\sigma = \dfrac{ne^2\tau / m}{1 - i\omega\tau}$ where *n* is the number of free carriers per unit volume in the conductor, *m* is the mass of these carriers, τ is their scattering time, and ω is the angular frequency. Obviously, the resistivity ρ is just the inverse of this expression. The appearance of the imaginary part of σ and ρ is due to the inertia of the charge carriers. In what follows you may assume that the frequency is so high that the conductivity is purely imaginary: $\sigma = \dfrac{ne^2\tau / m}{1 - i\omega\tau} \quad \rightarrow \quad ine^2 / m\omega$. (N.B. I am assuming a time dependence of $e^{-i\omega t}$.)

a) (10 points) In this ultra-high frequency limit show that it is possible for a wholly *longitudinal* electromagnetic wave to exist in the conductor. In other words, show that there exists a solution to Maxwell's equations for which the electric field is of the form $\boldsymbol{E} = E_0 \, \hat{z} \, e^{i(kz - \omega t)}$ with E_0 a constant. Show that the frequency ω is independent of wavevector *k* and is restricted to a single value. Find that value.

b) (5 points) What is the magnetic field associated with this wave?