

Modeling air mass flow into a football and resulting pressure after cooling

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Abstract: Realistic modeling of the rapid filling process for a football, neglected in current media coverage, can easily account for a “greater than 2 psi” (*ESPN*, 21 January 2015) drop in internal pressure from the moment immediately after a ball is inflated and checked by the referees prior to an NFL game to the point, over two hours later, when it is used during competition. In general, this analysis supports the hypothesis that the New England Patriots did not present the referees with underinflated footballs prior to the AFC Championship Game in which they defeated the Indianapolis Colts on 18 January 2015.

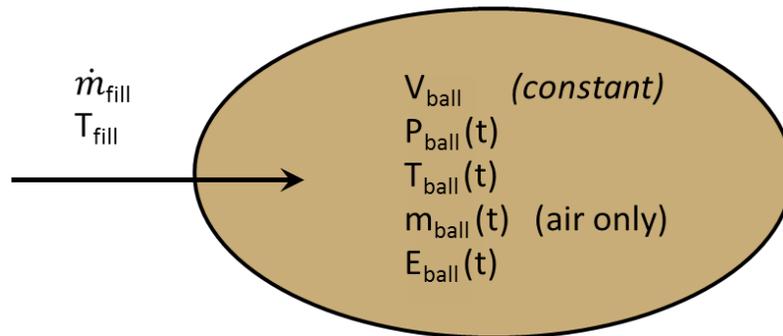


Figure 1. Schematic of the constant-volume control volume receiving the mass flow (a football), showing mass entering the ball and the known constants (ball volume V_{ball} and fill temperature and mass flow rate T_{fill} and \dot{m}_{fill}), the only directly measured quantities ($P_{ball}(t)$, where the “ t ” means that the quantity varies with time), and other quantities for the air inside the ball (not the leather ball itself; just the air on the inside) which also vary with time and may be calculated: temperature $T_{ball}(t)$, air mass $m_{ball}(t)$, and internal energy $E_{ball}(t)$.

A football filling with gas such as air (see Figure 1) can be modeled in three different ways, depending on heat transfer characteristics. If flow into the football has time to equilibrate with the walls and come to room temperature, the filling process can be considered as isothermal. If the flow occurs fast enough that heat transfer through the walls of the chamber is negligible, the filling process can be considered as adiabatic. For the case where heat transfer is finite, it must be included in the model with a convective heat transfer coefficient, h , through the walls of the football.

Leather is an excellent insulator and the filling process for a football, we may assume, happens relatively fast. We therefore model the ball adiabatically.

Adiabatic filling process

For the adiabatic assumption (essentially, that negligible heat is transferred out of the chamber being filled; for a leather ball filled quickly, this is probably a good assumption), T inside the ball is no longer constant at room temperature, even if constant temperature air is being injected. Neglecting kinetic energy, the change in internal energy E depends only on the energy (represented as specific enthalpy, h) of the mass flowing into the chamber, as heat transfer through the walls is assumed to be zero.

$$E = mc_v T \quad \text{and} \quad \frac{dE}{dt} = \dot{m} h_{fill} \quad \text{where} \quad h_{fill} = c_p T_{fill}$$

$$c_v T \frac{dm}{dt} + mc_v \frac{dT}{dt} = \frac{dm}{dt} c_p T_{fill}$$

Differentiating the expression for E , setting it equal to the expression for dE/dt , and gathering terms (noting that T is T_{ball}):

$$(c_v T_{ball} - c_p T_{fill}) \frac{dm}{dt} = -mc_v \frac{dT_{ball}}{dt}$$

And then using the ideal gas law to put dT/dt in terms of m and P , and simplifying by noting that for no change in volume, $dV/dt = 0$:

$$P = \frac{mRT}{V}$$

$$\frac{dP}{dt} = \frac{dm}{dt} \frac{RT}{V} + \frac{mR}{V} \frac{dT}{dt}$$

$$\frac{V}{mR} \frac{dP}{dt} - \frac{dm}{dt} \frac{T}{m} = \frac{dT}{dt}$$

Now substituting in the new expression for dT/dt into the expression for dT_{ball}/dt :

$$(c_v T_{ball} - c_p T_{fill}) \frac{dm}{dt} = -mc_v \left(\frac{V}{mR} \frac{dP}{dt} - \frac{dm}{dt} \frac{T}{m} \right)$$

$$c_v T_{ball} \frac{dm}{dt} - c_p T_{fill} \frac{dm}{dt} = c_v T_{ball} \frac{dm}{dt} - \frac{c_v V}{R} \frac{dP}{dt}$$

And cancelling the first term on each side, noting that the volume and pressure in the equation are for the ball, and solving for $dm/dt = \dot{m}$ we are left with:

$$\dot{m} = \frac{c_v}{c_p} \frac{V_{ball}}{RT_{fill}} \frac{dP_{ball}}{dt} \tag{1}$$

Therefore, if we had a recorded pressure trace for the football during the filling process (which could be differentiated to provide dP/dt) the mass flow over the filling process could be directly calculated. The ratio of specific heats c_p / c_v is well-known in fluid mechanics as γ , which for air under normal conditions is 1.4, and the inverse is therefore approximately 0.714. The volume of an American football is about 3.8L (or 0.0038 m³), the fill temperature is room temperature (75F or 297K), and the gas constant R for air is 287 J kg⁻¹ K⁻¹ (in this paper, all instances of R are the specific gas constant of air).

We do not actually have a recorded pressure trace for the football during the filling process. However, we can use Equation 1 above to figure out the mass change of the air in a football for a given pressure change by integrating both sides. For our purposes we need not model the time-dependent filling process; we only care about the beginning and ending pressures, and the amount (mass) of additional air it was necessary to put into the football to achieve that pressure rise, so Equation 1 becomes:

$$\Delta m = \frac{c_v}{c_p} \frac{V_{ball}}{RT_{fill}} \Delta P_{ball} \quad (2)$$

Atmospheric pressure is about 14.7 psi (absolute) at sea level, and the standard fill pressure for an NFL football is at minimum 12.5 psi (gauge), or 27.2 psi (absolute) at sea level, which is 187,500 Pascals (Pa) in metric units. Assume that the filling process begins with the footballs to be filled already pressurized a little higher than atmospheric pressure (they are unlikely to start completely deflated), but quite a bit lower than regulation pressure: say 15 psi (absolute), or 0.3 psi (gauge). In metric units this is 103,400 Pa. This also ensures that the volume of a ball changes only slightly with additional pressure, and for this analysis we'll therefore assume that the volume is constant. We'll also assume that the balls are at (locker) room temperature, as is the air they are to be filled with. The well-known Ideal Gas Law, Equation 3 below, can be used to calculate the quantity (mass) of air already inside the football:

$$PV_{ball} = mRT \quad \text{or,} \quad m = \frac{PV_{ball}}{RT} \quad (3)$$

Solving for m we find that the mass of the air already in the football is about 4.52 grams (g) or 0.00452 kg.

The amount of additional pressure (ΔP_{ball} in Equation 2) above to bring the football up to regulation pressure of 27.2 psi (absolute) is 12.2 psi (absolute), or 84,100 Pa. Using Equation 2, the amount of additional air (mass) to be added to the ball in an adiabatic (rapid) filling process is therefore about 2.68 g, for a total of 7.20 g of air in the football after the filling process, to the minimum regulation pressure, is complete.

Pressure after cooling

Equation 3, the Ideal Gas Law, can then be solved for P (with $m = 0.00720$ kg and a cold stadium temperature of 50F or 283K) to find the pressure at the end of an adiabatic filling process followed by a slow cooling to stadium temperature:

$$P_{ball} = \frac{mRT_{stadium}}{V_{ball}} = \frac{0.00720 \times 287 \times 283}{0.0038} = 153,900 \text{ Pa} = 7.6 \text{ psi (gauge)} \quad (4)$$

It should be noted that this is a “worst case” in terms of pressure drop resulting from the adiabatic filling effect, which assumes no heat flow at all out of the ball during the relatively short filling process, and a ball that is originally filled only to the NFL minimum pressure. The much simpler “best case”, as has been presented in the media, involves using the Ideal Gas Law after an assumed isothermal filling process (in other words, assuming either perfect heat flow out of the ball or a very long, slow filling process), which leaves a total of 8.36 g of air in the football. While this additional 1.16 grams of air would be undetectable in terms of the weight of the ball, it makes a big difference in pressure, as Equation 4 above becomes:

$$P_{ball} = \frac{mRT_{stadium}}{V_{ball}} = \frac{0.00836 \times 287 \times 283}{0.0038} = 178,700 \text{ Pa} = 11.2 \text{ psi (gauge)}$$

Conclusions

For footballs filled from a supply of room temperature air to the NFL minimum pressure of 12.5 psi (gauge), and then cooled for a period of time sufficient to bring the air inside the football to a temperature of 50F, analysis indicates that the pressure at the end of the cooling time would be expected to be **between 7.6 psi (gauge) and 11.2 psi (gauge)** due purely to atmospheric effects, with the exact value depending upon how rapidly the footballs were filled and the heat transfer properties of pebbled pigskin leather (which are not considered here). According to media reports, 11 out of 12 New England-provided footballs at the end of the AFC Championship Game were under pressure by “greater than 2 psi”, or in other words, were found with measured pressures of less than 10.5 psi (gauge). This value is consistent with the range of values that would be expected from the natural processes of thermodynamics.

This analysis makes it appear to be quite unlikely that the New England Patriots either underinflated their footballs initially or deflated them during the course of the game. At the very least, intentional deflation or underinflation have been shown to be unnecessary to explain the measured pressures post-game. It is much more likely that a combination of the filling process itself and the cold temperatures in the stadium resulted in the low measured pressures observed when the footballs were examined after the game. It is in fact likely that many cold-weather games have significantly underinflated footballs by the end of the match, especially in cases where the team providing the footballs chooses to initially inflate them to values near the

minimum allowed by the NFL. Furthermore, that the Colts' footballs were apparently found to be at higher pressure when checked—despite being exposed to similar atmospheric conditions—is also potentially innocuous and may indicate that 1) the initial fill pressure of the Colts' balls was closer to the 13.5 psi (gauge) upper limit; 2) the Colts' balls were filled more slowly than the Patriots' (which makes the filling process more isothermal as the temperature has more time to equilibrate during the process of adding air); 3) more time elapsed between the filling of the Colts' balls and their initial measurement; or some combination of all of these factors.

Implications and recommendations

A basic principle of thermodynamics holds that **one must define not one but *two* thermodynamic variables to completely specify the state of a gas system**. The NFL's standard, by specifying an acceptable range for the pressure of the balls only, fails to fully define the quantity of air that should be inside the football. Therefore, we are left with a situation where two teams which fill their balls differently—one quickly and one more slowly—or at different temperatures may be entirely within the rules of the game, and yet put very different quantities of air into the football.

I recommend that the NFL change their standard to specify a **pressure range at thermal equilibrium**, where equilibrium is established either at a temperature equivalent to the playing conditions on the field, or at some specified standard temperature (e.g. a typical room temperature). In practical terms, the procedure to achieve this could be as simple as checking the balls again once or twice during the two-hour cooling period before the game (where they are held at or near the standard temperature; either room temperature or the temperature outdoors), and topping them off with small amounts of additional air as necessary after these checks.

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Brief biography

Joe Jewell, a native of Stevensville, Michigan, graduated from the California Institute of Technology with a double major in Aeronautics and Medieval History, and completed a master's degree in Aerospace Engineering at the University of Michigan before winning the Rhodes Scholarship, which he used to earn a research master's degree in hypersonics at the University of Oxford in the UK. He defended his Aeronautics PhD at Caltech in May 2014, where he investigated high Mach number nonequilibrium flows in the T5 Hypervelocity Shock Tunnel at GALCIT, supported by the National Defense Science and Engineering Graduate Fellowship and the Jack Kent Cooke Foundation Graduate Scholarship. Joe is currently a postdoctoral researcher at the Air Force Research Laboratory at Wright-Patterson AFB, where he works through the Universal Technology Corporation.