

Ph125c, Homework #3
Due Tuesday April 24 at noon

1. Earlier this term we solved for bound states of a spherical square well. In this problem we'll look at scattering from a 'hard sphere' potential defined by

$$V(r) = +\infty \quad (r < a), \\ = 0 \quad (r > a).$$

Compute the differential and total scattering cross sections under the assumption that the energy of the incident particle

$$E_k = \frac{\hbar^2 k^2}{2m}$$

is so low that $ka \ll 1$, and hence that only the s -wave ($l = 0$) term contributes significantly in the partial wave expansion

$$f_k(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} \sqrt{4\pi(2l+1)} \exp(i\delta_l) \sin \delta_l Y_l^0(\theta).$$

Find the scattering phase shift δ_0 by explicitly solving the radial equation with $l = 0$, subject to the boundary condition $u_{k,0}(a) = 0$. Provide a geometric interpretation of the total cross section.

For questions 2 and 3, please have a look at the paper by R. Legere and K. Gibble, "Quantum Scattering in a Juggling Atomic Fountain" Phys. Rev. Lett. **81**, 5780 (1998), available in Millikan Library or from <http://prl.aps.org>.

2. Explain the sentence in the third paragraph on page 5780, "We vary the collision energy from low energies, where the scattering is s wave, since the de Broglie wavelength is much longer than the scale of the interatomic potential, up to energies at which several partial waves are allowed." What is the relevance of the de Broglie wavelength?

3. Equation (1) on page 5782 should look familiar, when you take into account the equivalence

$$Y_l^0(\theta) = \sqrt{\frac{2l+1}{4\pi}} P_l(\cos\theta).$$

In the first paragraph after the paragraph that contains Eq. (1), the authors state that the differential cross-section is sensitive to small p wave contributions because of the $s - p$ interference term. Explain.

4. Merzbacher, Chapter 19 Exercise 19.3, p. 486.

5. Consider a 1D linear harmonic oscillator with Hamiltonian

$$\mathbf{H}_0 = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega_0\mathbf{x}^2,$$

prepared initially in its ground state. In this problem we wish to consider depopulation of the ground state by time-dependent perturbations.

For a time-dependent perturbation of the form

$$V(t) = V_0\mathbf{x}\cos\omega t,$$

use the results of first-order time-dependent perturbation theory to argue that there is a resonance when $\omega = \omega_0$. What are the limitations of such an argument?

Using a similar approach, deduce the resonance condition for a perturbation of the form

$$V(t) = \Omega\mathbf{x}^2\cos\omega t,$$

which represents periodic modulation of the spring constant. Would such a perturbation have a similar effect on a classical harmonic oscillator, starting in its ground state?