Isotropy and Statistics of the CMB Anisotropy Maps – *Planck* 2018 Results

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Pasadena CA

on behalf of the Planck Collaboration

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Why Study Isotropy and Statistics of the CMB?

1. Isotropy (i.e. uniformity in all orientations) is a cardinal property of the universe at large scales. It is best probed by the angular distribution of the fluctuations in temperature and polarization of the cosmic microwave background (CMB) radiation. As an empirical fact, CMB isotropy strongly motivates the cosmological principle, and shapes theoretical ideas, e.g. inflation. Due to these fundamental implications it is essential to quantify the statistical isotropy of the CMB at all scales.
2. Statistical distribution of the primordial CMB fluctuations is empirically determined (thus far, most accurately by *Planck*) to be very nearly Gaussian. This is in accord with the simplest inflationary scenarios for generation of primordial perturbations. Hence, any significant findings of deviation from Gaussianity in the statistics of the measured CMB anisotropies is usually considered an indicator of a possible presence of foreground residuals and/or secondary anisotropies. If such candidate effects can be accounted for, or eliminated, then physics beyond the standard cosmological model might be invoked to explain convincing measurements.
3. At a practical level, isotropy and Gaussianity are usually assumed in the empirical derivation of the CMB anisotropy power spectra and the determination of cosmological parameters.
4. Extensive studies of Isotropy and Statistics of the CMB in the *Planck* temperature anisotropy data were reported previously (in 2013 and 2015) . Presently, we report the extension of these studies to the final release of data by the *Planck* collaboration.
5. Our current focus is on the full sky measurements of polarization anisotropies of the CMB. Given the present state of the measurements, this is a very challenging endeavor.

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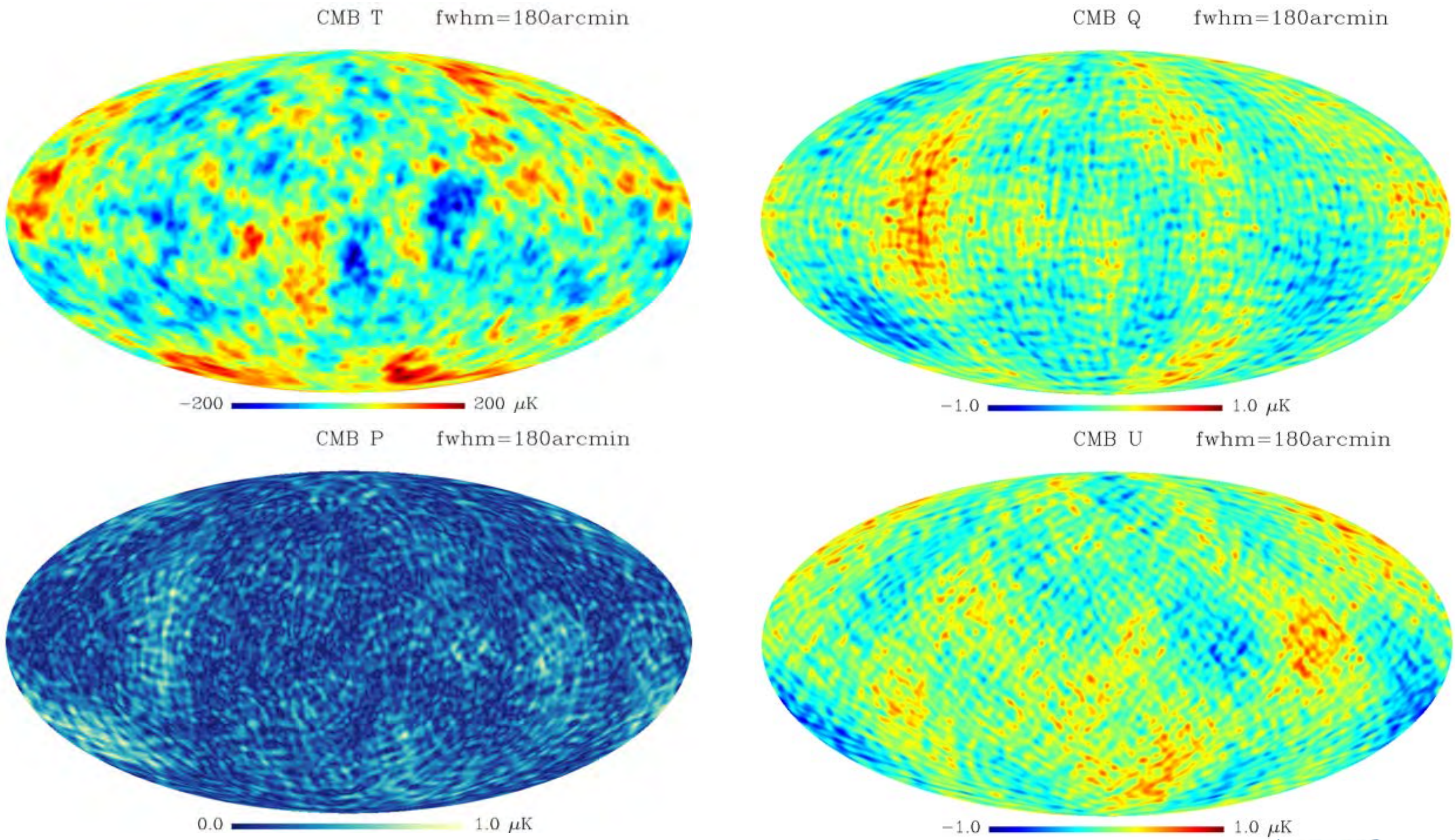
Planck 2018 results. VII. – in preparation

Desiderata
or
What would we like to see ...

planck

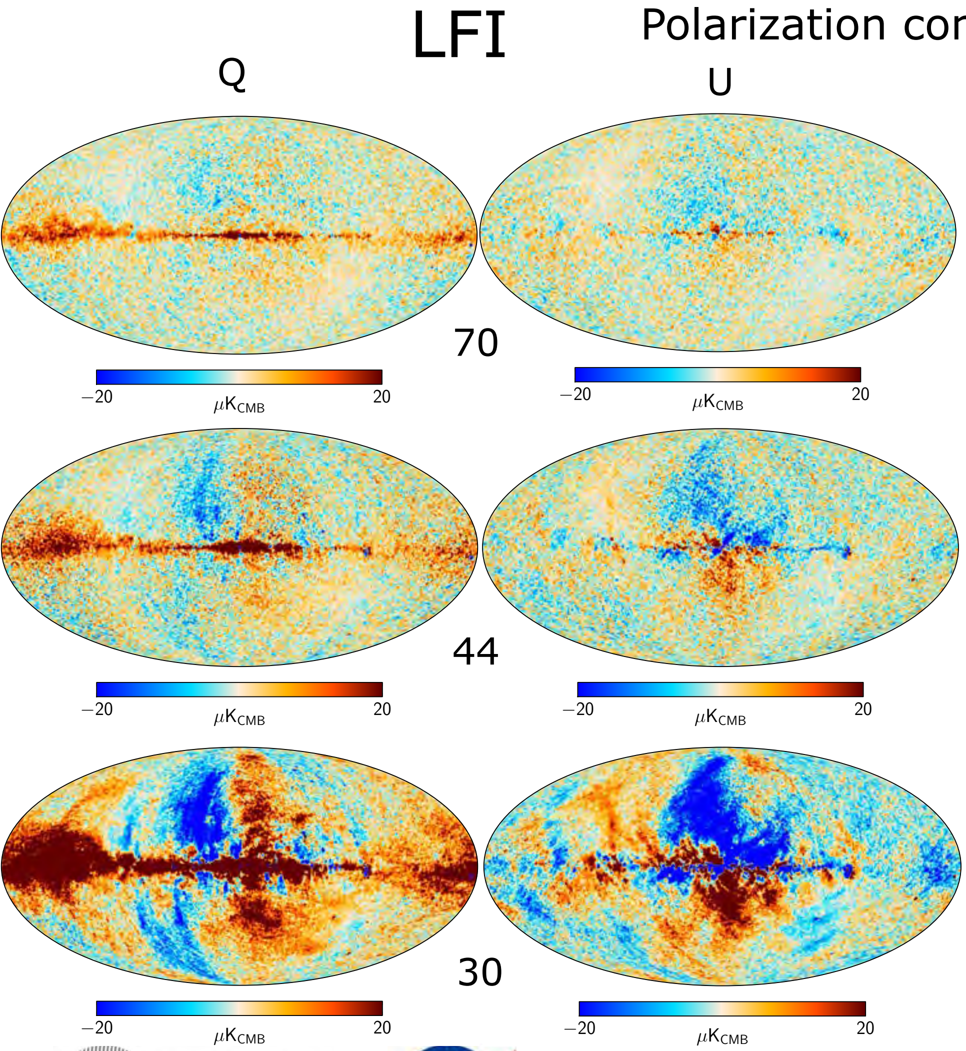
A random realization of

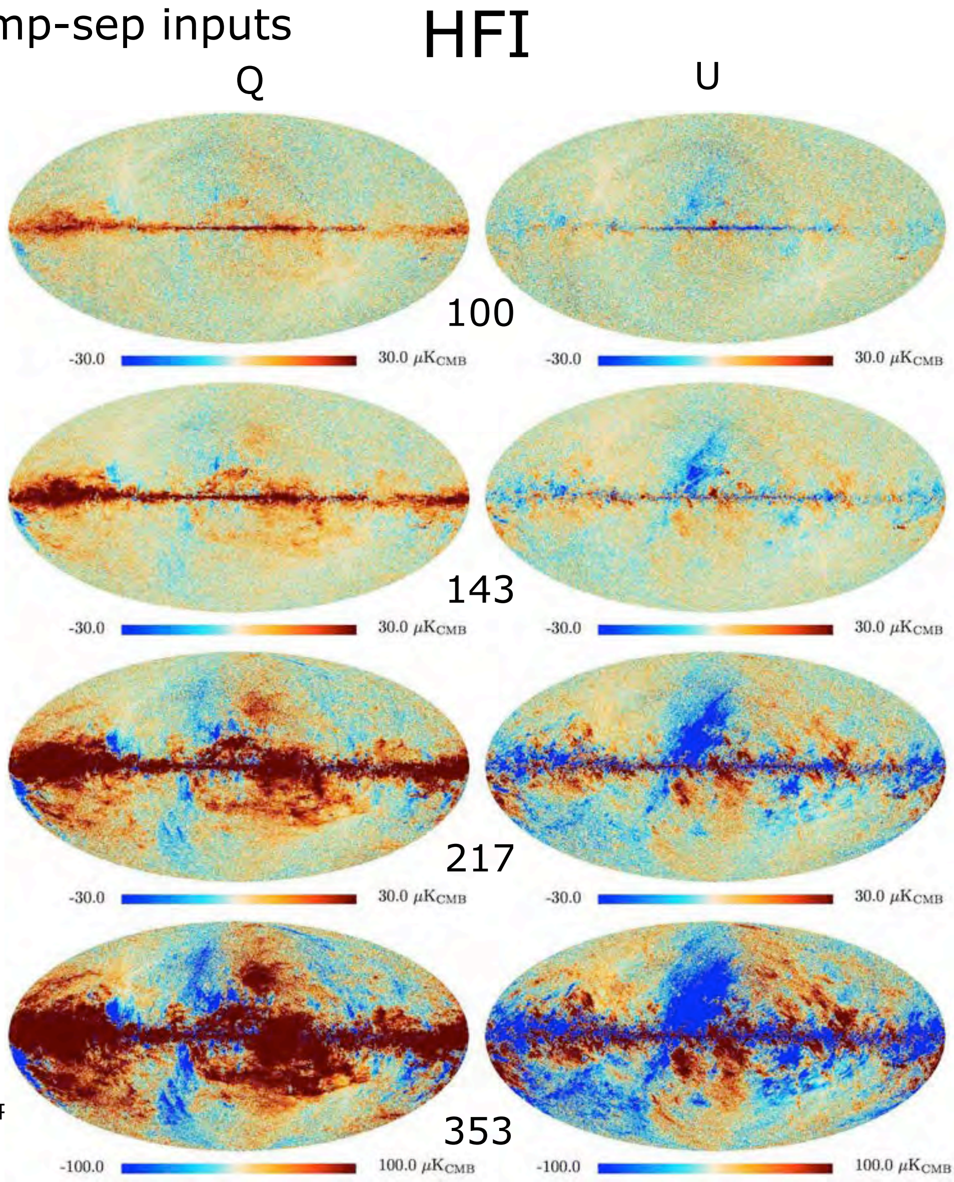
Noise-free CMB anisotropies in Planck-normalized ΛCDM cosmological model; smoothed to fwhm=**180** arcmin



Planck 2018 LFI and HFI Q and U frequency maps

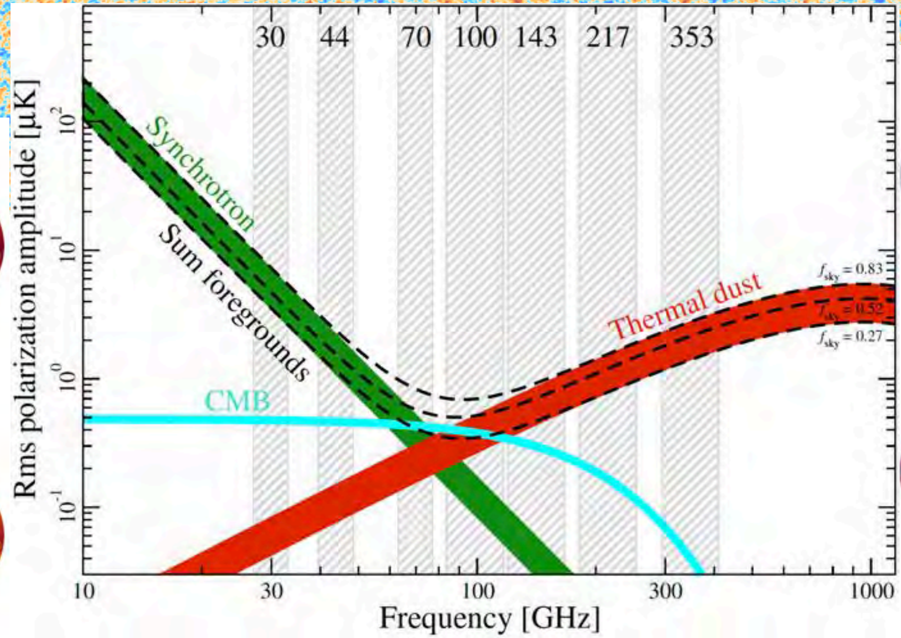
Polarization comp-sep inputs

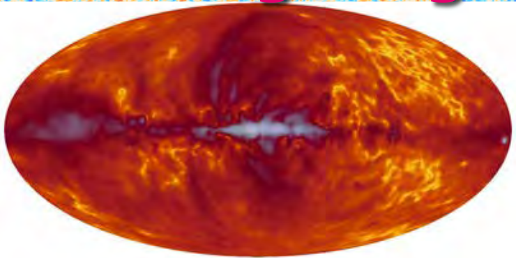




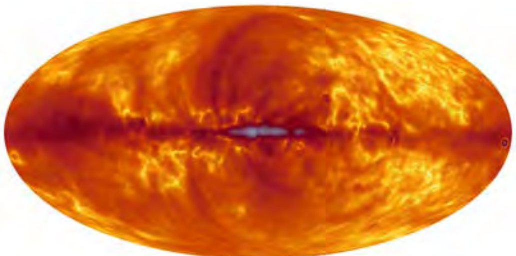
K.M. Górski, COSPAR

What are we fighting?

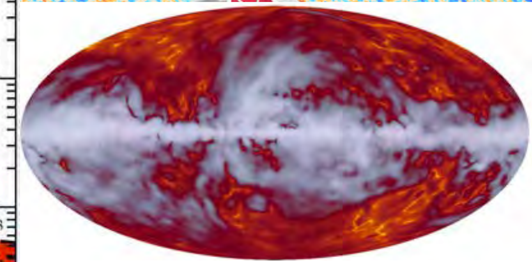




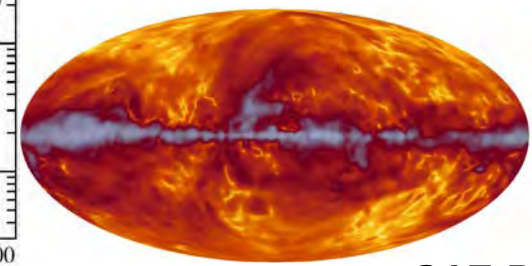
30 P



44 P

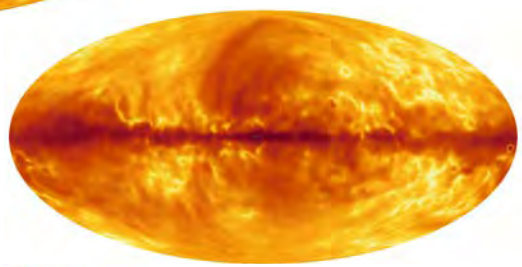


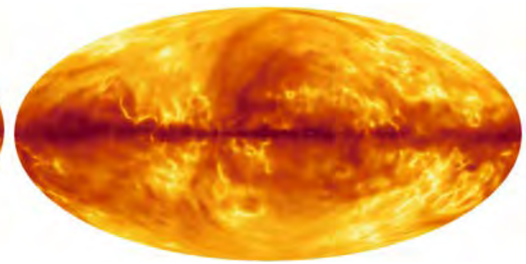
353 P

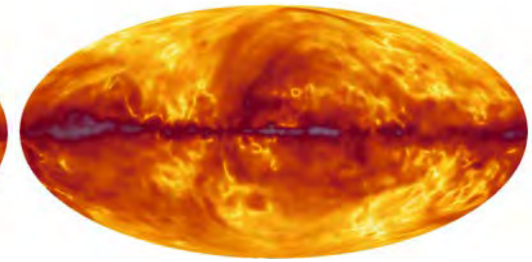


217 P

ALL PLOTS:
Polarization
Amplitude



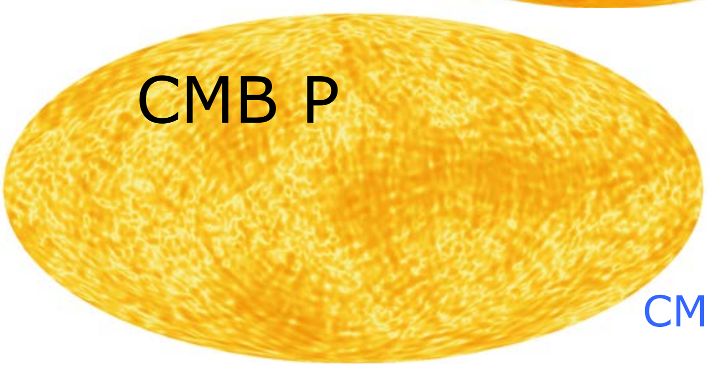




70 P

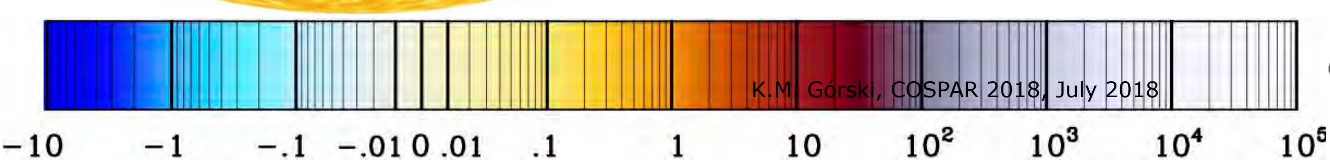
100 P

143 P



All maps at FWHM = 3deg

CMB - a realization from the τ =0.06 and r=0.10 model



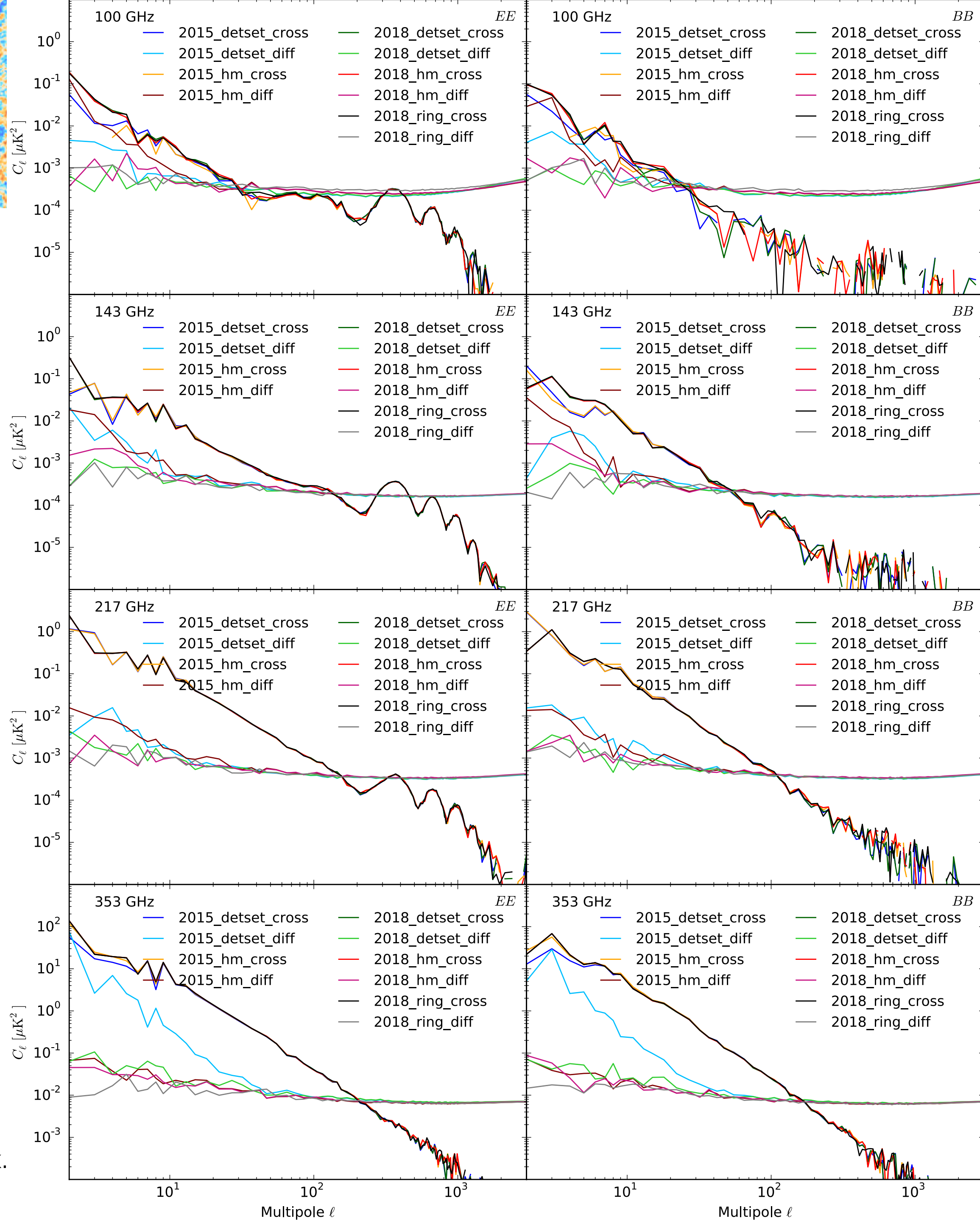
$\delta T\ [\mu K_{CMB}]$

What are we fighting; spectral picture ...

Double whammy:

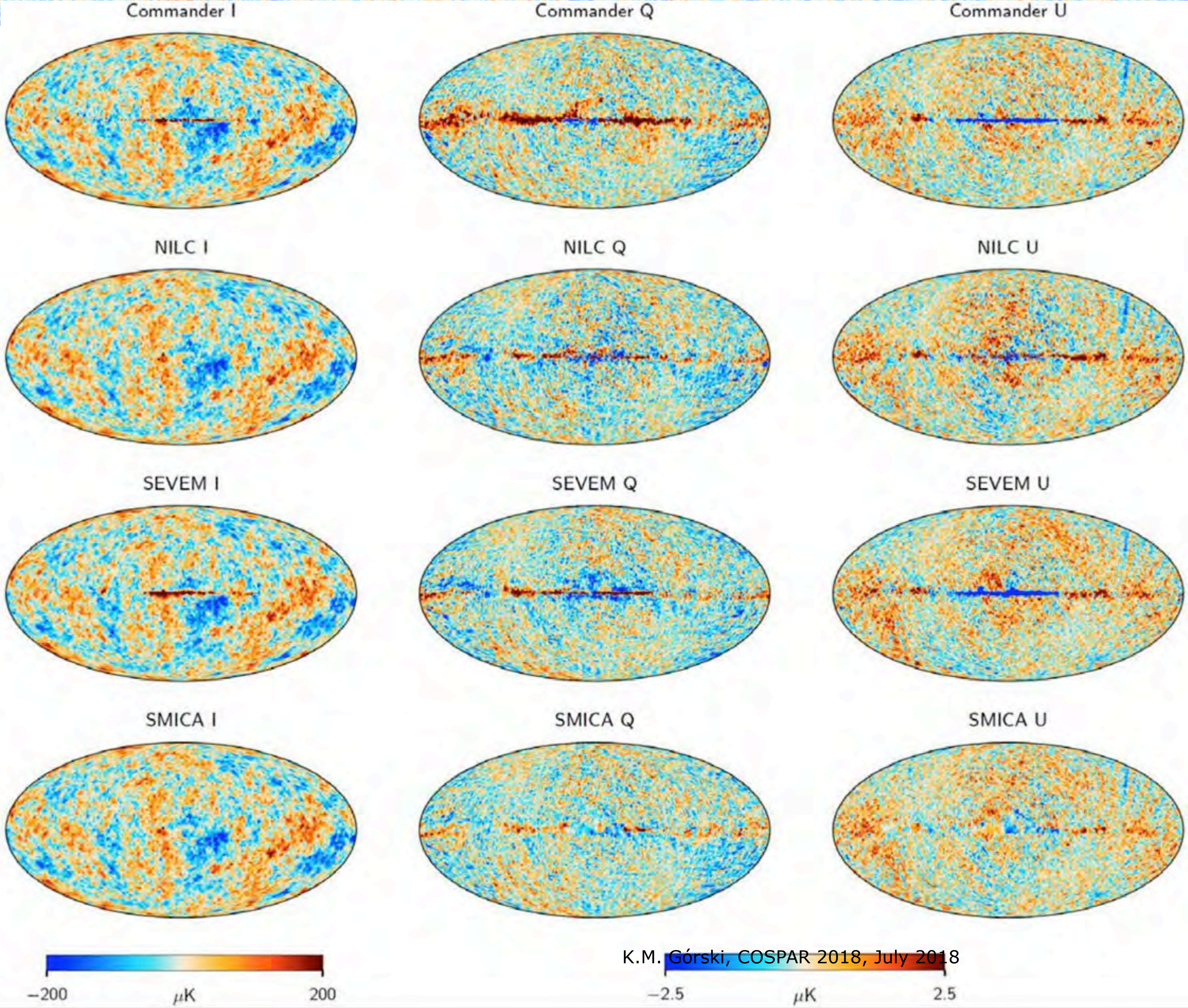
low-\ell foregrounds

and high-\ell noise



K.

Planck 2018 IQU CMB maps derived by four component separation methods



These are our inputs to Isotropy and Statistics studies

We work with both the Q and U maps as well as with their transformation into the E and B maps

Spectra of the *Planck* 2018 comp-sep polarization maps

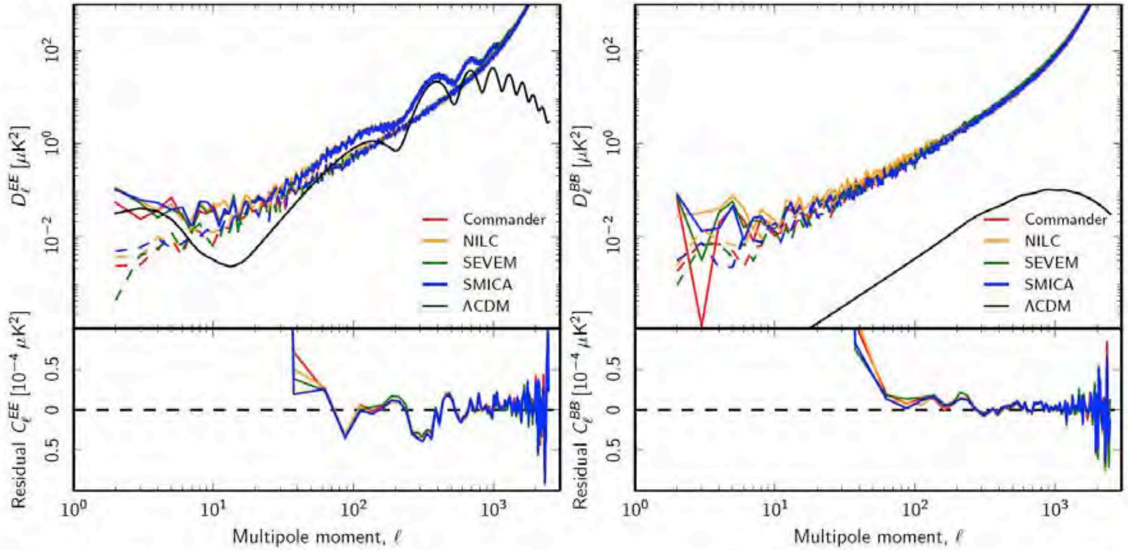


Fig. 16. Comparison of half-mission polarization power spectra. The top panel shows the half-sum (solid lines) and half-difference (dashed lines) power spectra, while the bottom panel shows the difference between the half-sum and the best-fit *Planck* 2018 ΛCDM and half-difference spectra. The latter residual spectrum is binned with $\Delta\ell = 25$.

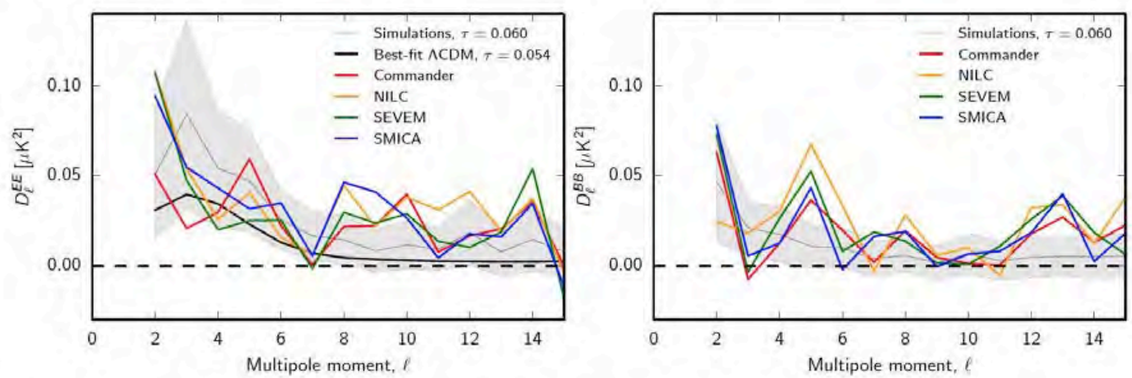
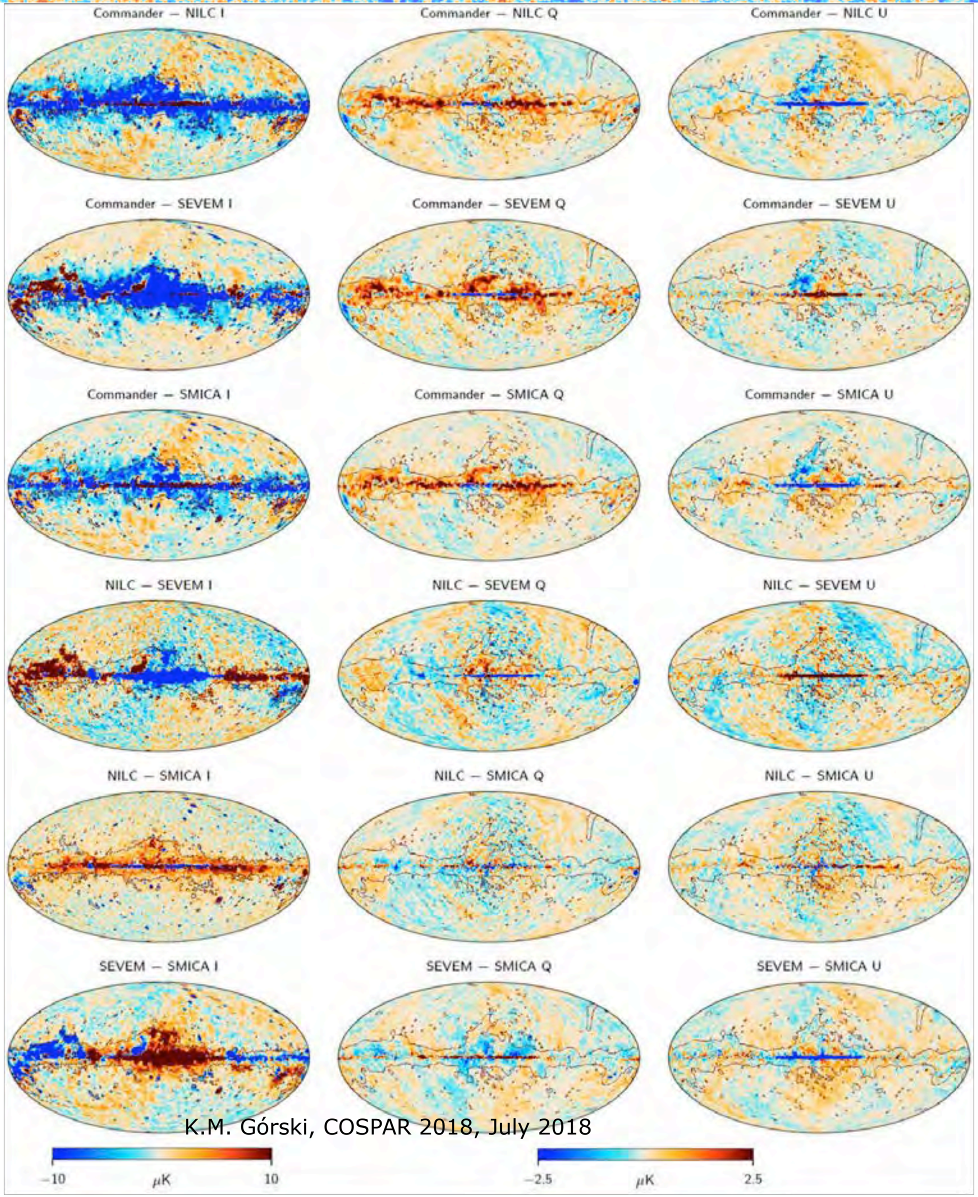


Fig. 17. Comparison of low-ℓ half-mission polarization EE (*left*) and BB (*right*) power spectra. Each spectrum is computed as the difference between the respective half-sum and the half-difference spectrum. Grey bands show 1σ confidence regions derived from 300 end-to-end simulations as processed by Commander. Formally speaking, these can therefore only be directly compared with the red curve. Note that the value for the optical depth of reionization adopted for these simulations is $\tau = 0.060$ (see Planck Collaboration II 2018; Planck Collaboration III 2018 for details), which is larger than the best-fit *Planck* 2018 value of $\tau = 0.054$.

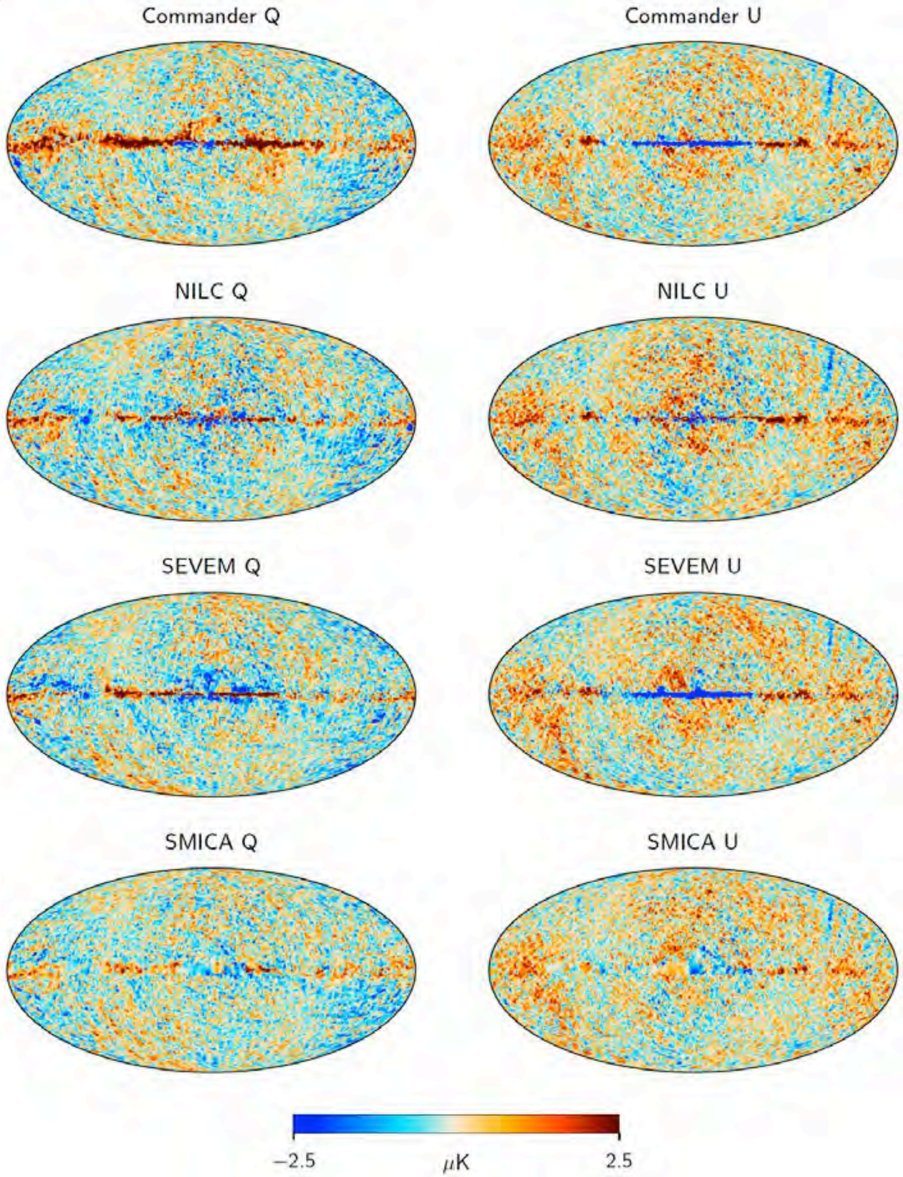
Fidelity of the *Planck* 2018 component separation solutions for CMB I Q U maps

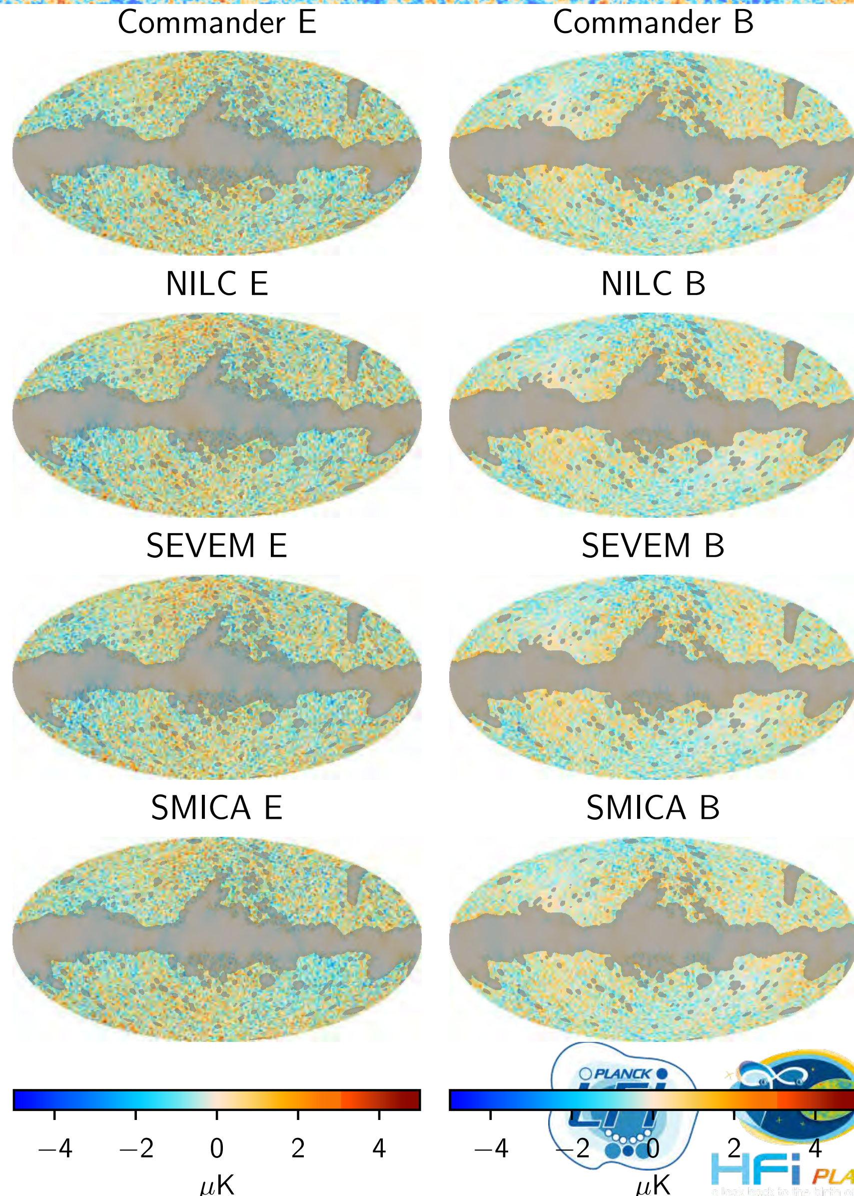
4 comp-sep methods – 6 pair-wise difference maps.

All difference Q/U maps show large scale features of amplitude exceeding that of cosmological signals

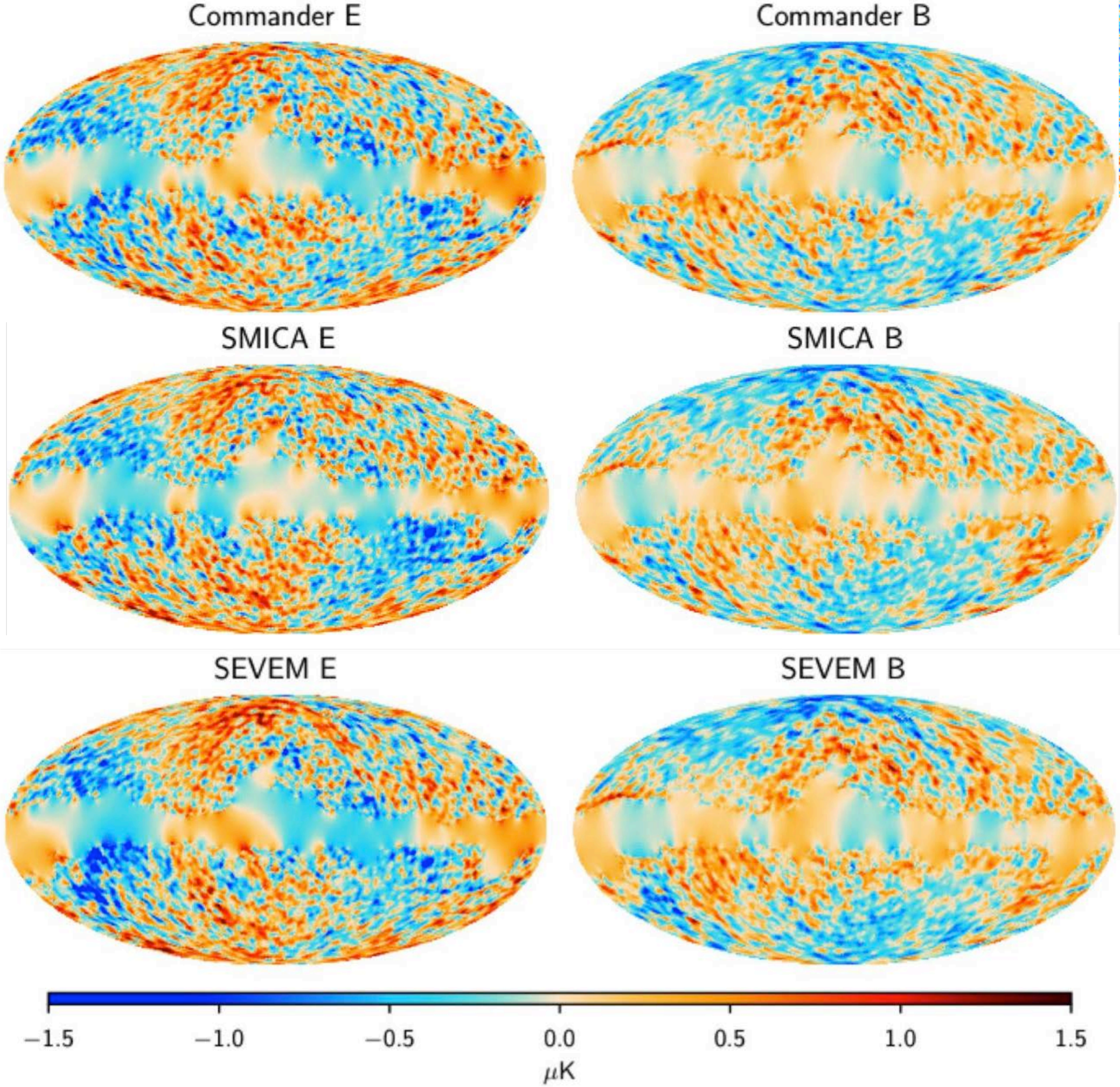


E- and B-mode Reconstruction





Low resolution E/B maps



A few examples of what we have studied thus far ...

1. Tests of non-Gaussianity
   a. 1-D moments
   b. N-point correlation functions
   c. Minkowski functionals
   d. Peak statistics
   e. Stacking of CMB peaks
2. Anomalies in the microwave sky
   a. Deficiency of large-angle correlations
   b. Hemispherical asymmetry
   c. Point-parity asymmetry
   d. The Cold Spot and other large-scale features in T and polarization
3. Dipole modulation and directionality
   a. Variance asymmetry
   b. Dipole modulation and QML analysis
   c. Angular clustering of the power distribution

All quantitative assessments of these results are built against a MC simulation ensemble of 999 CMB sky and 300 noise+systematics realizations.

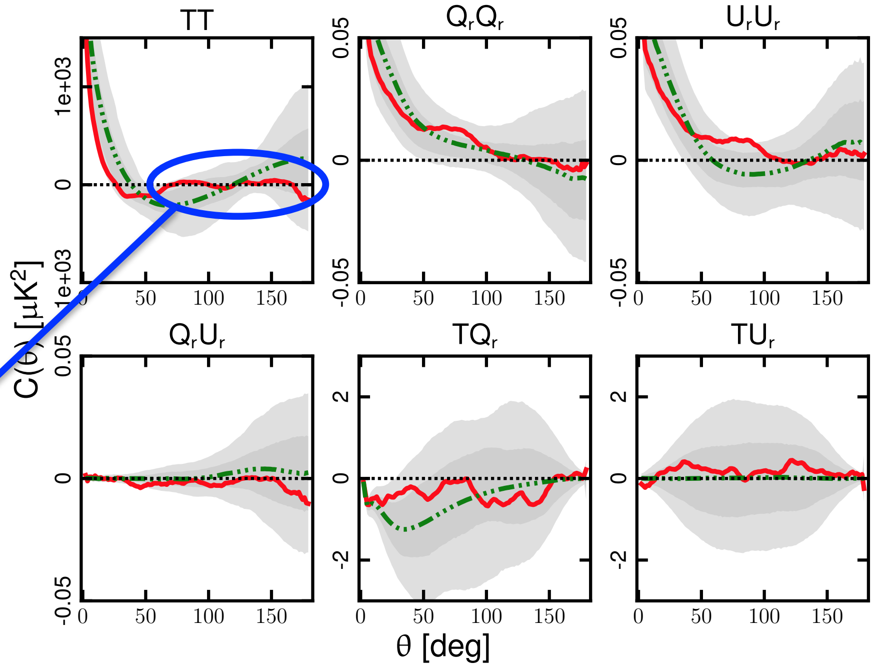
N-point correlation functions for *Planck* CMB maps: Large-angle correlations

- Testing lack of correlations using statistics

$$S^{XY}(\theta_1,\theta_2) = \int_{\cos\theta_2}^{\cos\theta_1} \left[\hat{C}_2^{XY}(\theta)\right]^2 d(\cos\theta)$$

Table 12. Global *p*-value for the $S^{XY}(\theta, 180°)$ statistic for the *Planck* fiducial ΛCDM model at most as large as the observed values of the statistic for the *Planck* 2018 CMB maps with resolution parameter $N_{\rm side} = 64$.

Statistic	Probability [%]			
	Comm.	NILC	SEVEM	SMICA
TT	98.5	98.5	98.5	98.5
Q_rQ_r	76.2	79.9	48.3	51.5
U_rU_r	76.0	77.9	47.5	50.2
TQ_r	88.0	95.1	85.9	95.3
TU_r	68.9	64.1	59.0	46.5
Q_rU_r	43.1	4.0	51.1	45.8



- Consistency of the N-point functions between data and simulations for ΔT, Q and U maps
- No statistically significant deviation of the CMB maps from Gaussianity
- Lack of large-angle correlations in the case of the CMB temperature maps
- No anomalous correlations at large-angles in the case of the CMB polarization maps

Minkowski Functionals

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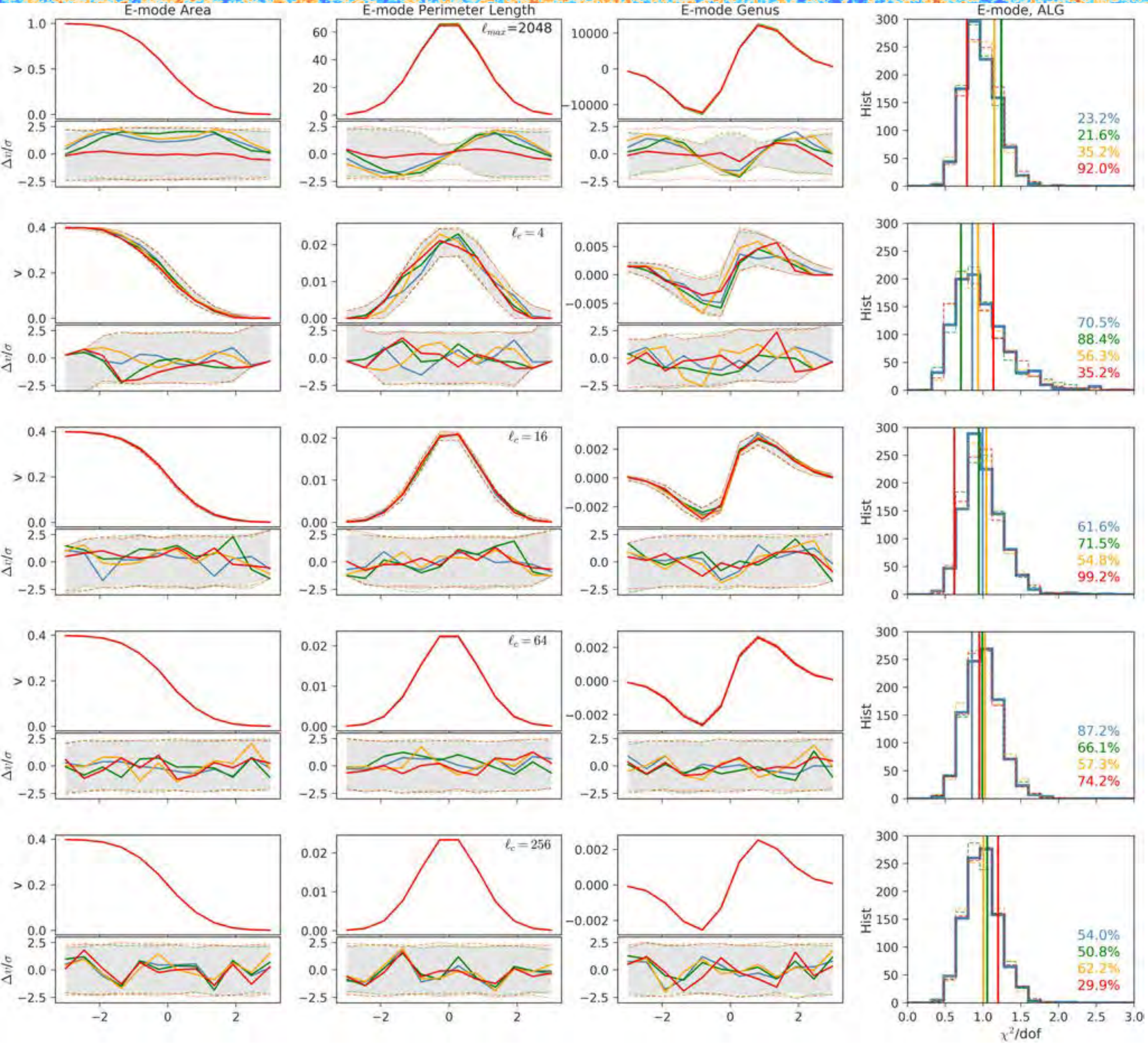
1. MFs of the random field f (x) and its excursion sets A over threshold *u*
   a. $V_0(A_u(f))$ is the area of the excursion regions, or the first Minkowski functional.

   b. $V_1(A_u(f))$ is half the boundary length of the excursion regions, or the second Minkowski functional.

   c. $V_2(A_u(f))$ is the genus or the Euler-Poincarè characteristic (minima+maxima-saddles) of the excursion regions, or the third Minkowski functional.

Minkowski Functionals: E-mode results



1. Temperature data agrees well with the FFP10 simulations
2. The E-mode polarization data shows a good agreement between data and simulation albeit inconsistencies exist among different component separation methods.

(needlet space results)

Statistics of E and B Peaks

Gaussian peak statistics [Bond and Efstathiou, 1987]:

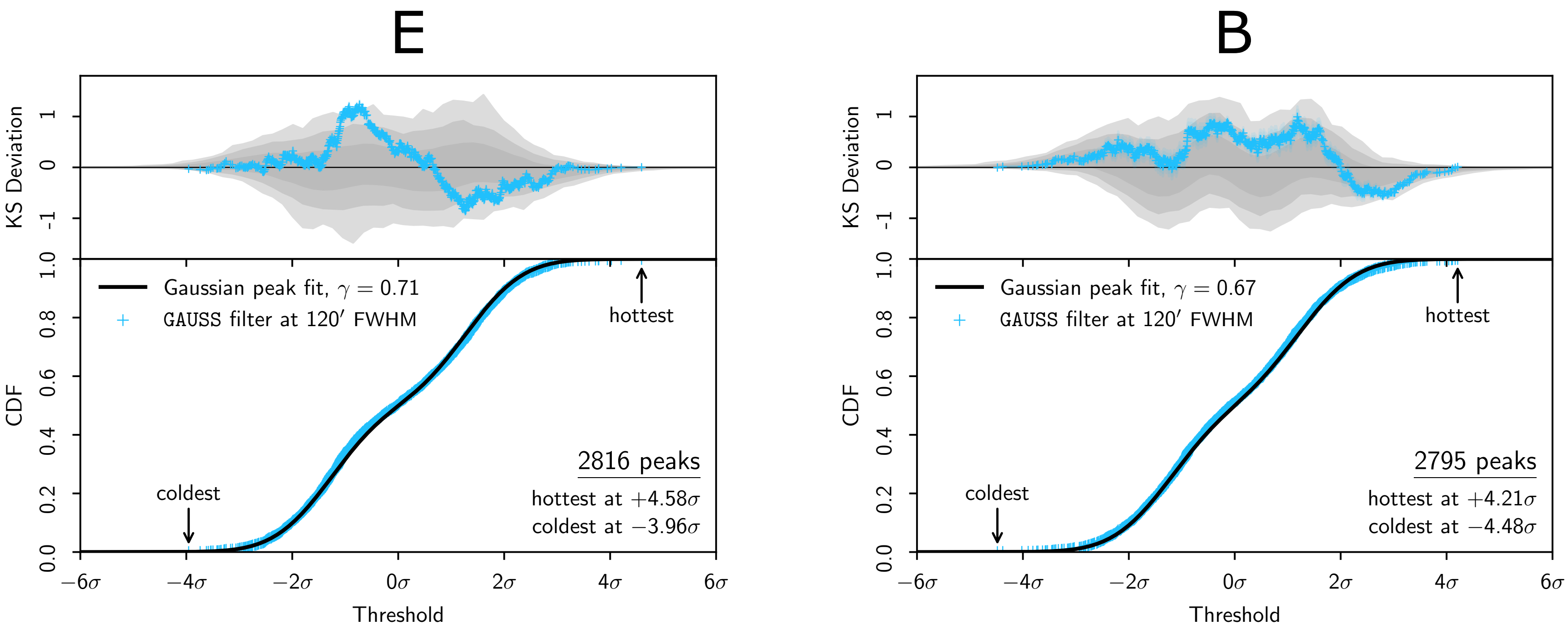
$$\frac{n_{\rm max} + n_{\rm min}}{n_{\rm pk}} \left(\frac{x}{\sigma} > \nu \right) = \sqrt{\frac{3}{2\pi}} \gamma^2 \nu \exp\left(-\frac{\nu^2}{2} \right) + \frac{1}{2} \,{\rm erfc} \left[\frac{\nu}{\sqrt{2 - \frac{4}{3}\gamma^2}} \right]$$

Characterize deviation by Kolmogorov-Smirnov test:

$$D_n = \sup_x |F_n(x) - F(x)|, \quad {\rm Probability}(D_n > {\rm observed}) = Q_{\rm KS}\left(n_{\rm eff}^{1/2} D_n \right)$$

(maximal distance between CDFs of two distributions)

Statistics of E and B Peaks

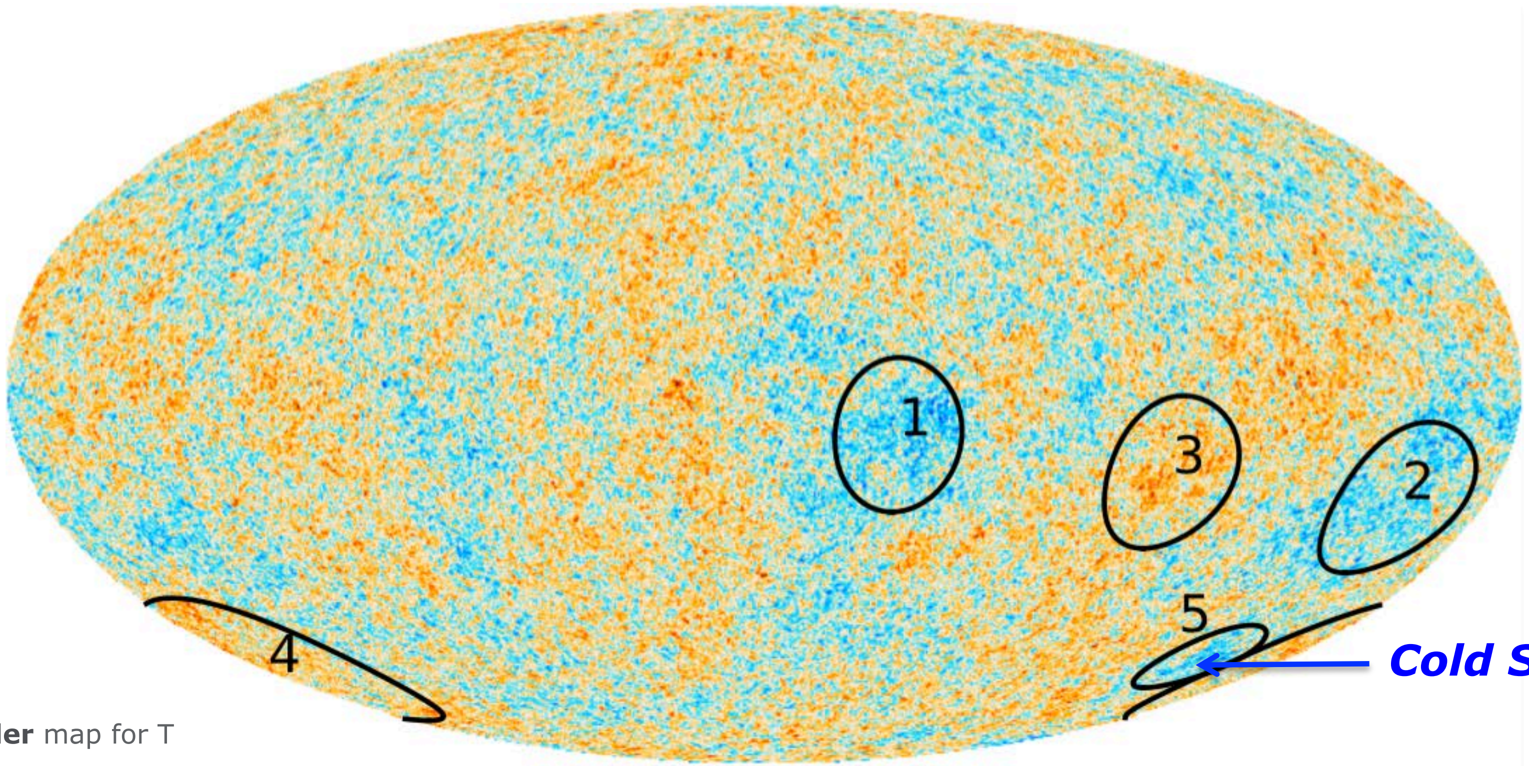


SMICA E and B maps, pre-whitened
smoothed with 120’ FWHM Gaussian

The *Cold Spot* and other large-scale peaks

The ***Cold Spot*** is an anomalous CMB feature of **large area** and a **very negative amplitude**, and a **large kurtosis** at scales of around **5 deg**.

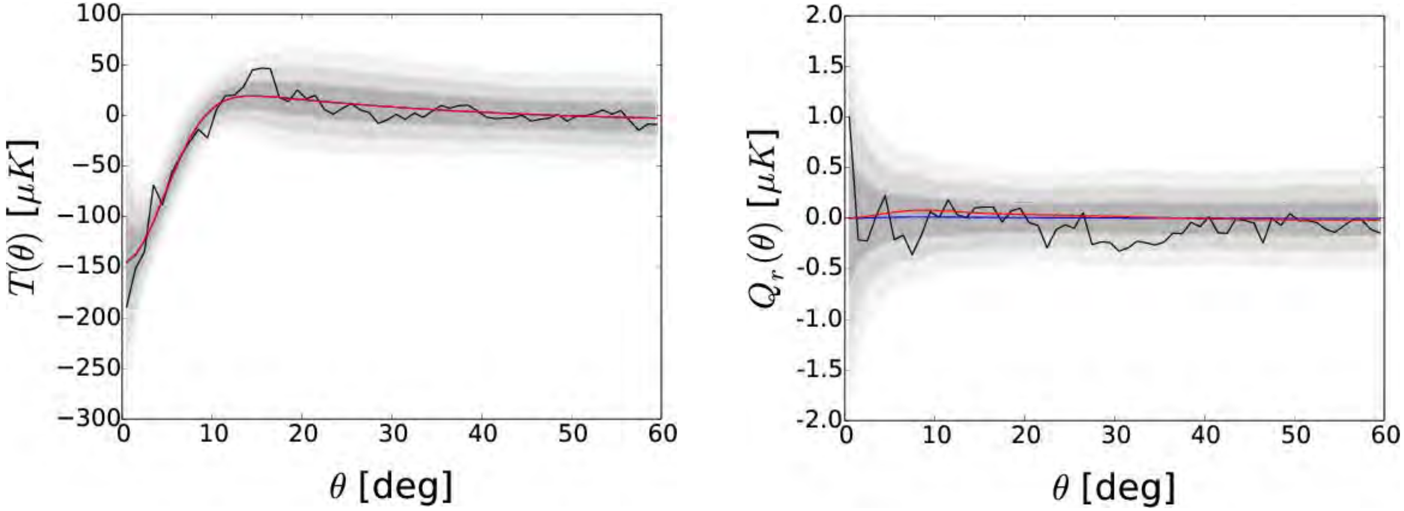
Besides the ***Cold Spot*** we have also investigated the multipolar profiles of **four more large-scale peaks**, which have been previously identified anomalous features at very large scales (at 10 deg).



Commander map for T

The *Cold Spot* and other large-scale peaks

The ***$T_0(\theta)$*** and ***$Q_{r,0}(\theta)$*** **profiles** for the ***Cold Spot*** as estimated from the CMB SEVEM map (similar results for the other Planck CMB maps). Shaded regions represent 1, 2 and 3 σ confidence levels. **Dark line** is the **measured profile**, whereas the **red** one is the **theoretical value**.



One could also allow a **fit to the observed profiles**, by introducing a **free normalization parameter** (A) for the theoretical profiles ***$Q_r(\theta)$***.
A=1 would imply a perfect **agreement with ΛCDM**. A value of **A close to 0** would be an indication of **lack of polarization** wrt ΛCDM.

The *Cold Spot* and other large-scale peaks

Convincing determination of the lack of polarization pattern in accord with the temperature profile, **in the case of the *Cold Spot***, could be interpreted as a **secondary anisotropy origin** for that fluctuation.

Results

Polarization

Peak	Comm.	NILC	SEVEM	SMICA
Peak 1	-1.330 ± 1.102	-0.200 ± 1.146	0.164 ± 1.117	-0.034 ± 1.134
Peak 2	0.978 ± 1.088	1.523 ± 1.088	0.446 ± 1.124	0.791 ± 1.077
Peak 3	0.129 ± 1.261	0.090 ± 1.298	0.196 ± 1.270	-0.331 ± 1.311
Peak 4	0.691 ± 0.957	0.150 ± 1.011	1.981 ± 0.994	0.516 ± 0.927
Peak 5	-0.232 ± 0.951	-0.170 ± 1.035	0.152 ± 1.019	-0.598 ± 1.023

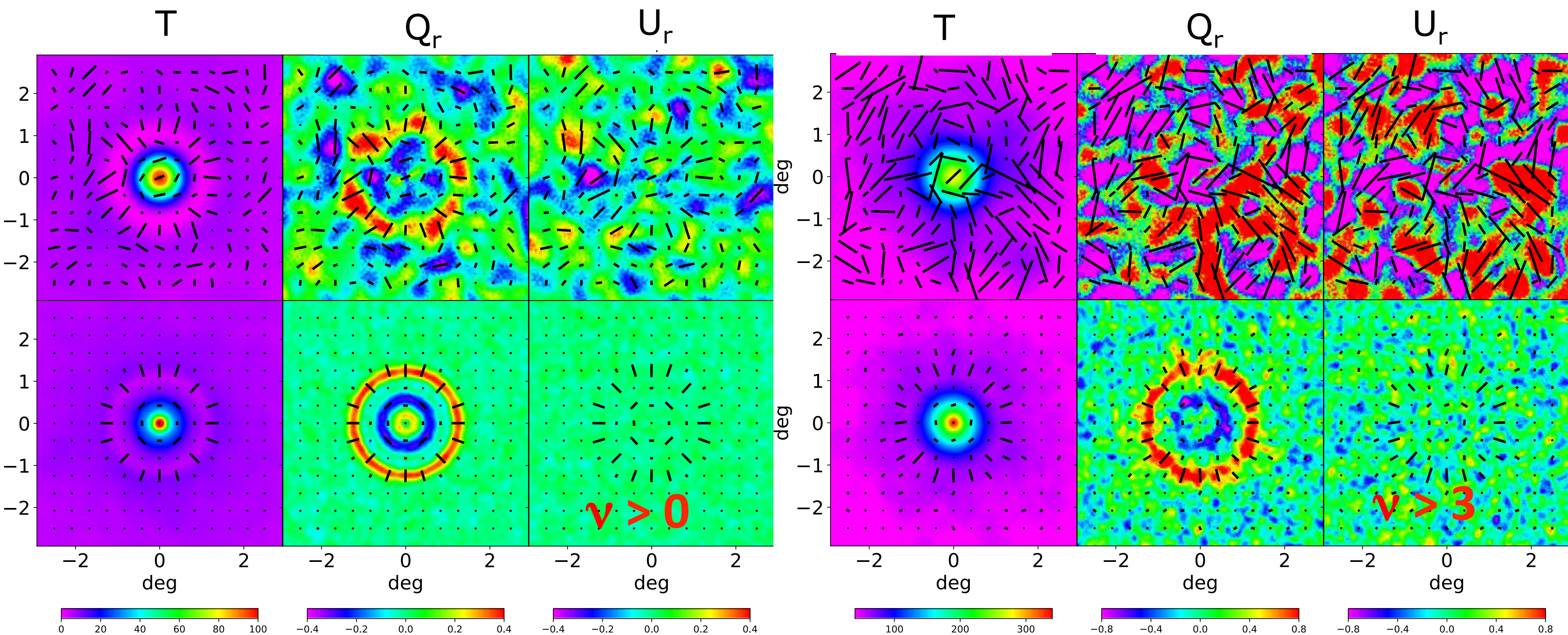
In **temperature**, the peak profiles are in agreement with ΛCDM, whereas for **polarization**, the **noise is too large to obtain significant result**.

Non-oriented stacking

- CMB peaks selected on the intensity map at N_{side}=1024 above a certain threshold ν (in dispersion units of the map).
- The Q_r pattern traces the radial and tangent polarisation around the peaks predicted by ΛCDM due to the acoustic oscillations.
- The U_r component acts as a consistency test.

Planck-WMAP(V+W) comparison

Planck sensitivity and resolution allow to show **more defined stacking patterns** than **WMAP**.



WMAP: V+W band at N_{side} = 512 and 30′ (top rows)

Planck: SEVEM map at N_{side} = 1024 and 10′ (bottom rows)

Summary

1. Analysis of the *Planck* 2018 data set indicates that the statistical properties of the **CMB temperature anisotropies are in excellent agreement with previous studies**, as we see
   a. consistency with the Gaussian predictions of the ΛCDM cosmological model, and
   b. confirmation of the presence of anomalies on large angular scales.
2. The current study is a novel attempt at a comprehensive analysis of the statistics of the polarization signal over all angular scales, using either maps of the Stokes parameters Q/U, or the corresponding E-mode signal.
3. Our studies are limited both by our understanding of the data, and the corresponding ability to simulate the residual systematics present therein, the amplitude of which can be comparable to the cosmological signal.
4. Null tests applied to the polarization maps indicate that these issues do not dominate the analysis on angular scales of ~1 deg and larger.
5. In this regime, we do not detect cosmological non-Gaussianity, or anomalies corresponding to those found in temperature.
6. Notably, the stacking of CMB polarization signals centered on the temperature hot and cold spots exhibits excellent agreement with the ΛCDM cosmological model, and demonstrates that *Planck* provides state-of-the-art measurements of CMB anisotropies in temperature and polarization on degree scales.

The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.