Planck constraints on inflation

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on behalf of the Planck Collaboration

The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada

Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.

Inflation as seen by Planck

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The full sky measurement of CMB anisotropies down to a resolution of 5‘ with a noise sensitivity of 0.6 muK per deg allowed Planck to measure the power spectrum of temperature, polarization, CMB lensing and the bispectrum at high sensitivity and unprecedent extent in multipole range.

Either Planck noise sensitivity and the range of multipoles covered (which in temperature covers essentially the whole range in which CMB primary fluctuations dominate over foreground residuals and secondary anisotropies) have been of key importance for the constraints on inflation.

Key predictions of the simplest inflationary models, i.e. standard single field slow-roll models, were sufficient to explain the PR1 and PR2 data.

Theoretical interpretations motivated by inflation for intriguing anomalies on the largest angular scales (such as low-l deficit at $l \leqq 40$, feature at $l \approx 20$) and at high multipoles were not required at a statistically significant level.

Planck 2018 results on inflation

This highlights of *Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211, L10 of this release will be covered as a list of key questions relevant for inflation:*

What is the value of the scalar tilt n_s ?

Does n_s depend on the wavelength?

Is the Universe spatially flat?

Are tensor modes required?

Which inflationary models are best able to account for the data?

What model-independent constraints can be placed on the primordial power spectrum?

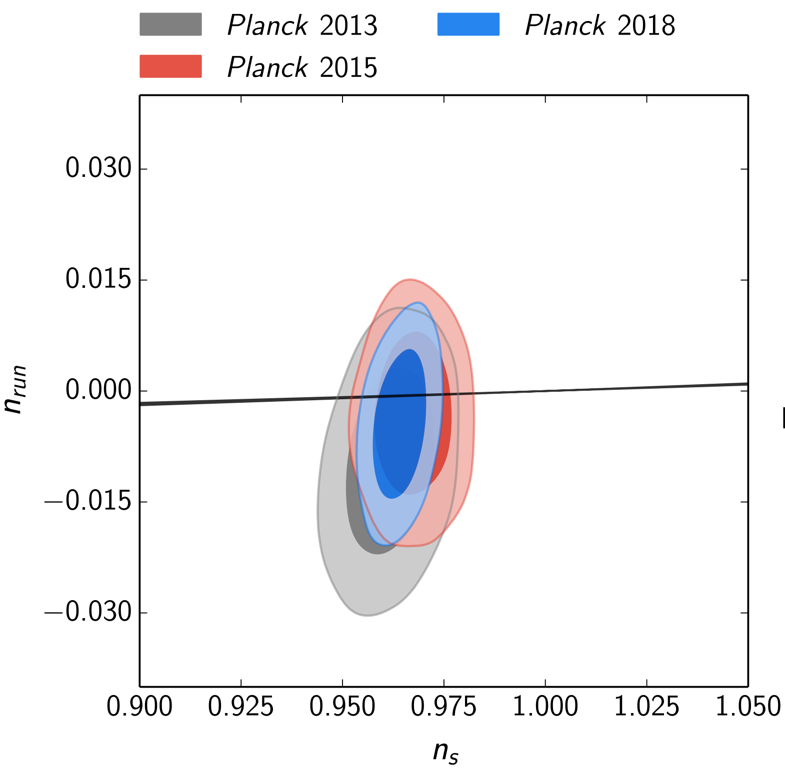
Is there evidence for features in the primordial power spectrum?

Were the primordial cosmological perturbations solely adiabatic?

Were the primordial fluctuations statistically isotropic?

Focus on Planck data alone (Planck TT, TE, EE + lowE + lensing is the 2018 baseline) supplemented by the BICEP2/Keck Array B-mode likelihood (BICEP2/Keck Array collaboration, Ade et al. 2016) and a compilation of Baryonic Acoustic Oscillation (BAO) measurements when relevant.

Does n_s depend on the wavelength?



$$n_s = 0.9649 \pm 0.0042 \quad (68\,\%\ \text{CL},\ \textit{Planck}\ \text{TT,TE,EE+lowE+lensing})$$

Improvement in the uncertainties by a factor 1/3 with respect to Planck 2015 baseline:

$$n_s = 0.9677 \pm 0.0060 \quad (68\,\%\ \text{CL},\ \textit{Planck}\ 2015\ \text{TT+lowP+lensing})$$

No running of the scalar tilt:

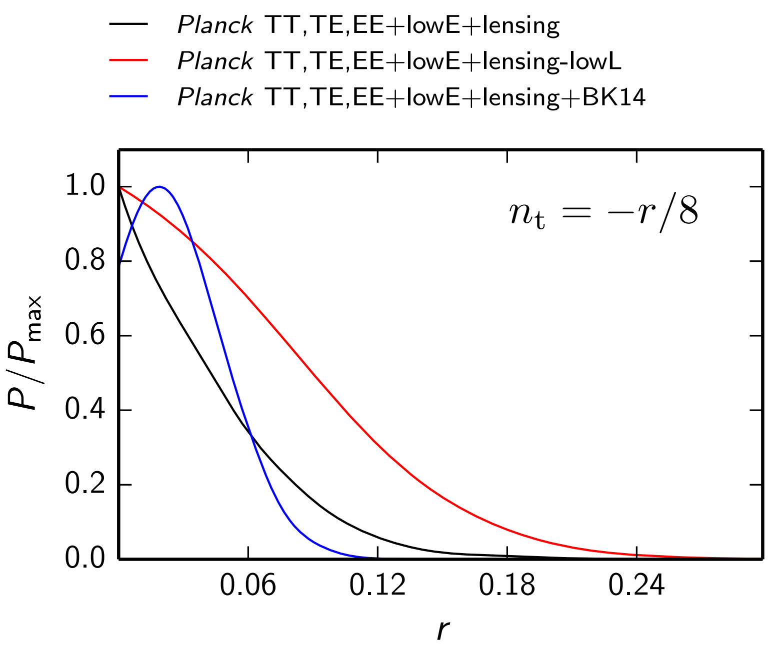
$$dn_s/d\ln k = -0.0045 \pm 0.0067 \quad (68\,\%\ \text{CL},\ \textit{Planck}\ \text{TT,TE,EE+lowE+lensing})$$

Planck 2018 high-ell polarization data put also tight constraints on the running of the running. For Planck TT (TT,TE,EE)+lowE+lensing we have:

$$dn_s/d\ln k = 0.013 \pm 0.012\ (0.002 \pm 0.010)$$
$$d^2 n_s/d\ln k^2 = 0.022 \pm 0.012\ (0.010 \pm 0.013)$$

No third or fourth derivatives of the potential are required from Planck data.

Are tensor modes required?



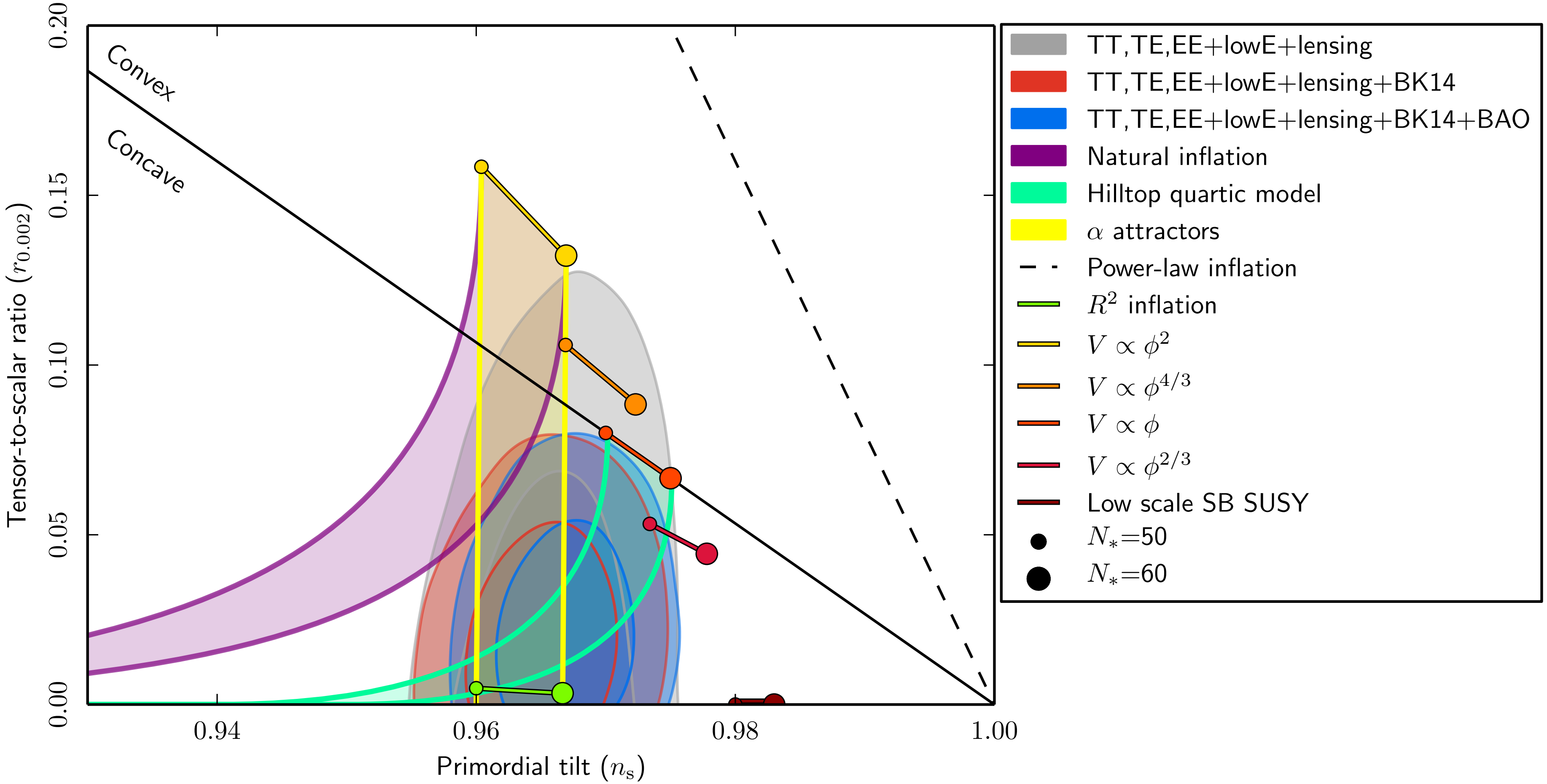
$$r_{0.002} < 0.10 \quad (95\,\%\ \mathrm{CL},\ \textit{Planck}\ \mathrm{TT,TE,EE{+}lowE{+}lensing})$$

$$r_{0.002} < 0.16 \quad (95\,\%\ \mathrm{CL},\ \textit{Planck}\ \mathrm{high}-\ell\ \mathrm{TT,TE,EE}\ \mathrm{+lowE{+}lensing})$$

$$r_{0.002} < 0.064 \quad (95\,\%\ \mathrm{CL},\ \textit{Planck}\ \mathrm{TT,TE,EE{+}lowE{+}lensing}\ +\ \mathrm{BK14})$$

$$V_* = \frac{3\pi^2 A_s}{2}\, r\, M_{\rm Pl}^4 < (1.7 \times 10^{16}\ \mathrm{GeV})^4 \quad (95\,\%\ \mathrm{CL})$$

Which inflationary models are best able to account the data?



Which inflationary models are best able to account the data?

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Inflationary model	Potential $V(\phi)$	Parameter range	$\Delta\chi^2$	$\ln B$
$R + R^2/(6M^2)$	$\Lambda^4\left(1 - e^{-\sqrt{2/3}\phi/M_{Pl}}\right)^2$	...	...	...
Power-law potential	$\lambda M_{Pl}^{10/3}\phi^{2/3}$	...	2.8	−2.6
Power-law potential	$\lambda M_{Pl}^3\phi$	...	2.5	−1.9
Power-law potential	$\lambda M_{Pl}^{8/3}\phi^{4/3}$	...	10.4	−4.5
Power-law potential	$\lambda M_{Pl}^2\phi^2$	...	22.3	−7.1
Power-law potential	$\lambda M_{Pl}\phi^3$	...	40.9	−19.2
Power-law potential	$\lambda\phi^4$	...	89.1	−33.3
Non-minimal coupling	$\lambda^4\phi^4 + \xi\phi^2 R/2$	$-4 < \log_{10}\xi < 4$	3.1	−1.6
Natural inflation	$\Lambda^4\left[1 + \cos(\phi/f)\right]$	$0.3 < \log_{10}(f/M_{Pl}) < 2.5$	9.4	−4.2
Hilltop quadratic model	$\Lambda^4\left(1 - \phi^2/\mu_2^2 + \dots\right)$	$0.3 < \log_{10}(\mu_2/M_{Pl}) < 4.85$	1.7	−2.0
Hilltop quartic model	$\Lambda^4\left(1 - \phi^4/\mu_4^4 + \dots\right)$	$-2 < \log_{10}(\mu_4/M_{Pl}) < 2$	−0.3	−1.4
D-brane inflation ($p = 2$)	$\Lambda^4\left(1 - \mu_{D2}^2/\phi^p + \dots\right)$	$-6 < \log_{10}(\mu_{D2}/M_{Pl}) < 0.3$	−2.3	1.6
D-brane inflation ($p = 4$)	$\Lambda^4\left(1 - \mu_{D4}^4/\phi^p + \dots\right)$	$-6 < \log_{10}(\mu_{D4}/M_{Pl}) < 0.3$	−2.2	0.8
Potential with exponential tails	$\Lambda^4\left[1 - \exp(-q\phi/M_{Pl}) + \dots\right]$	$-3 < \log_{10} q < 3$	−0.5	−1.0
Spontaneously broken SUSY	$\Lambda^4\left[1 + \alpha_h \log(\phi/M_{Pl}) + \dots\right]$	$-2.5 < \log_{10}\alpha_h < 1$	9.0	−5.0
E-model ($n = 1$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_1^E}M_{Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_1^E < 4$	0.2	−1.0
E-model ($n = 2$)	$\Lambda^4\left\{1 - \exp\left[-\sqrt{2}\phi\left(\sqrt{3\alpha_2^E}M_{Pl}\right)^{-1}\right]\right\}^{2n}$	$-2 < \log_{10}\alpha_2^E < 4$	−0.1	0.7
T-model ($m = 1$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_1^T}M_{Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_1^T < 4$	−0.1	0.1
T-model ($m = 2$)	$\Lambda^4\tanh^{2m}\left[\phi\left(\sqrt{6\alpha_2^T}M_{Pl}\right)^{-1}\right]$	$-2 < \log_{10}\alpha_2^T < 4$	−0.4	0.1

Table 5. Bayesian comparison for a selection of slow-roll inflationary models with w_{int} fixed (see text for more details). We quote 0.3 as the error on the Bayes factor. Models are strongly disfavoured when $\ln B < -5$.

Bayesian comparison between inflationary models taking into account the uncertainties in the reheating stage after inflation:

Slow-roll inflationary models as R2, D-brane inflation, alpha attractors, hilltop, potential wi exponential tails ... provide a similar fit to Planck and BK14 data.

Models with a monomial potential with a power larger or equal to 2 or with a potential including logarithmic correction (SB SUSY) are strongly disfavoured by Planck 2018 + BK14 data.

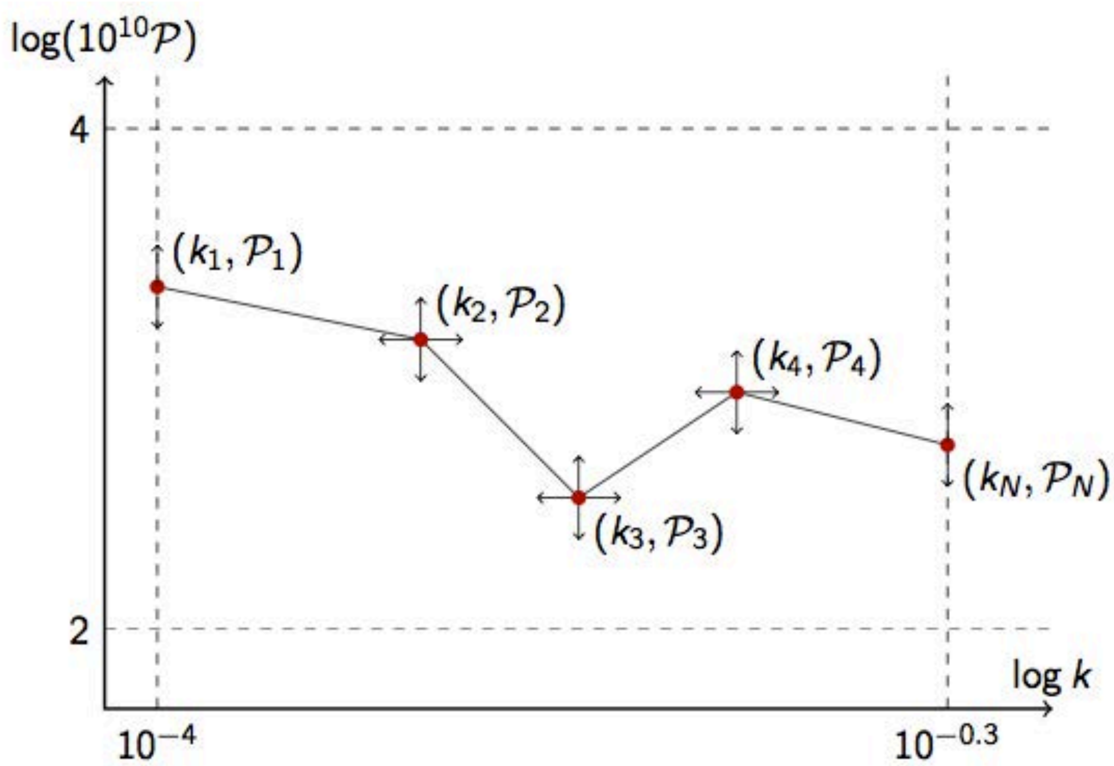
What model-independent constraints can be placed on the PPS?

Penalized likelihood

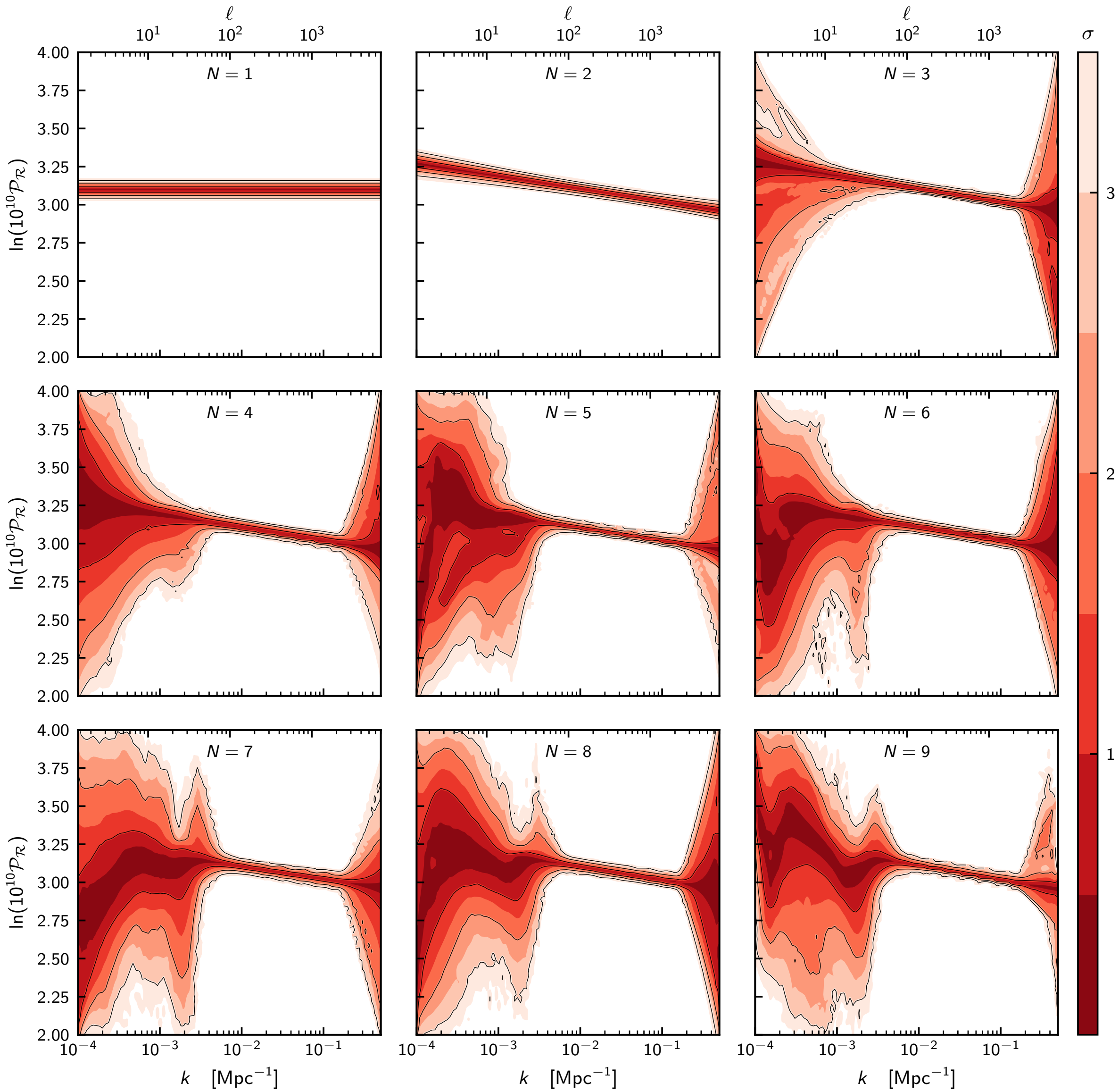
Bayesian reconstruction (plus analysis with alternative priors wrt 2015)

Cubic spline reconstruction

What model-independent constraints can be placed on the PPS?



What model-independent constraints can be placed on the PPS?



Log priors on movable nots

Planck 2018
TT, TE, EE + lowE + lensing

What model-independent constraints can be placed on the PPS?

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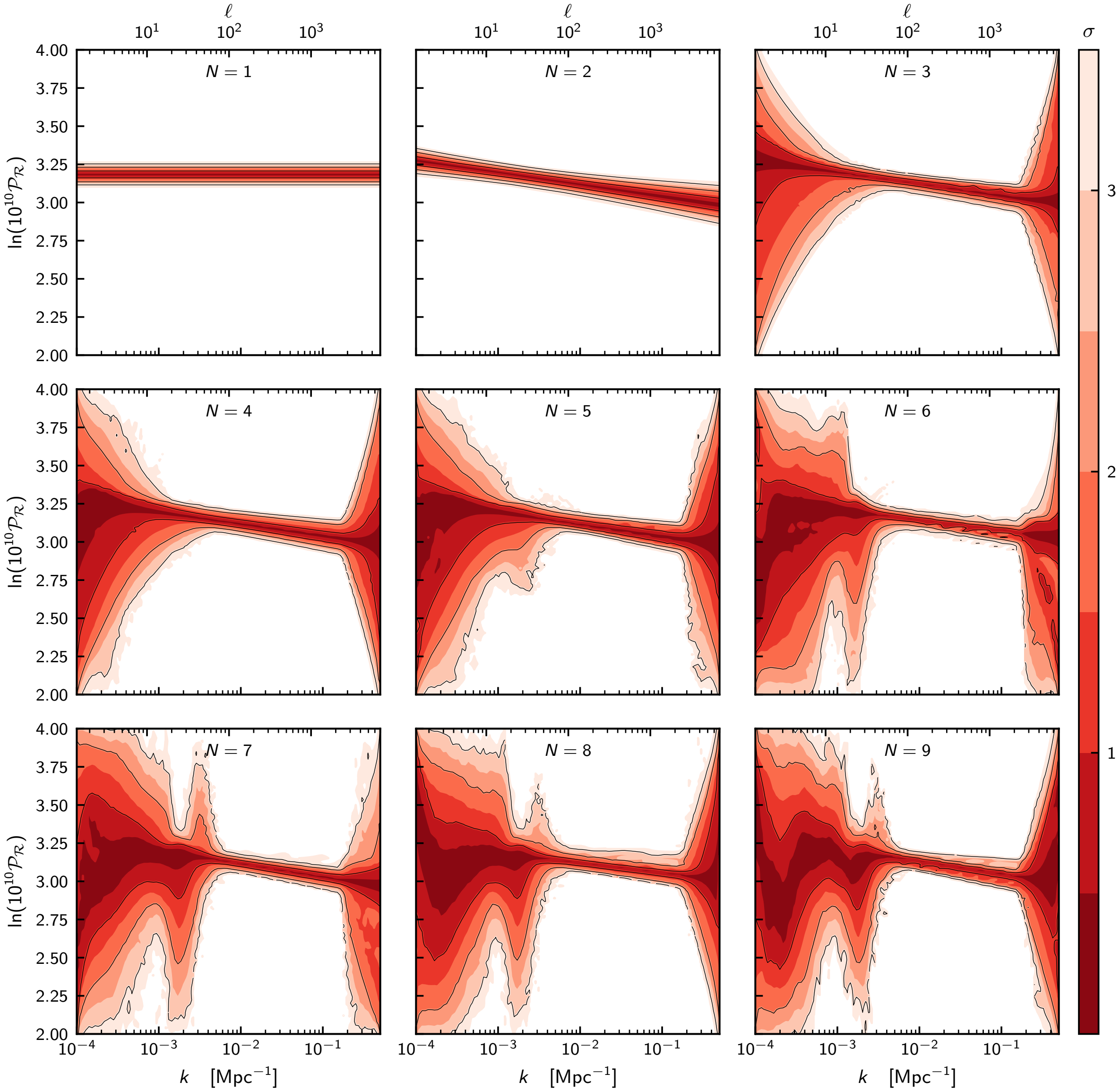
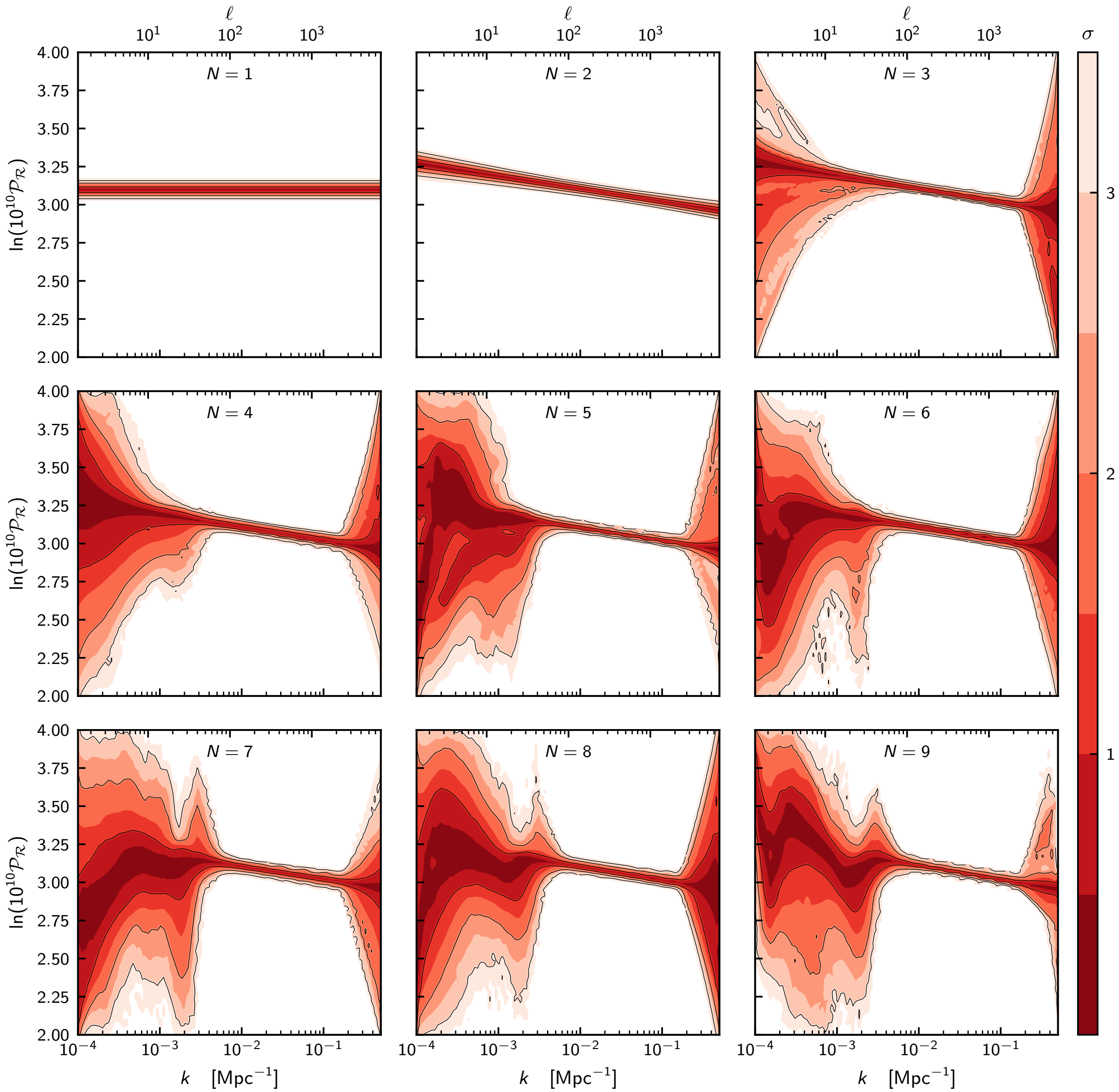


Log priors on movable nots

Planck 2015 TT + lowP

What model-independent constraints can be placed on the PPS?



Log priors on movable nots

Planck 2018
TT, TE, EE + lowE + lensing

What model-independent constraints can be placed on the PPS?

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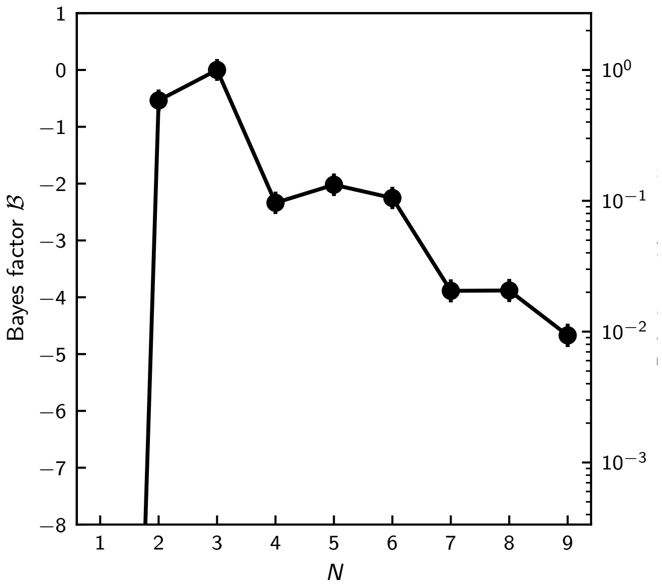
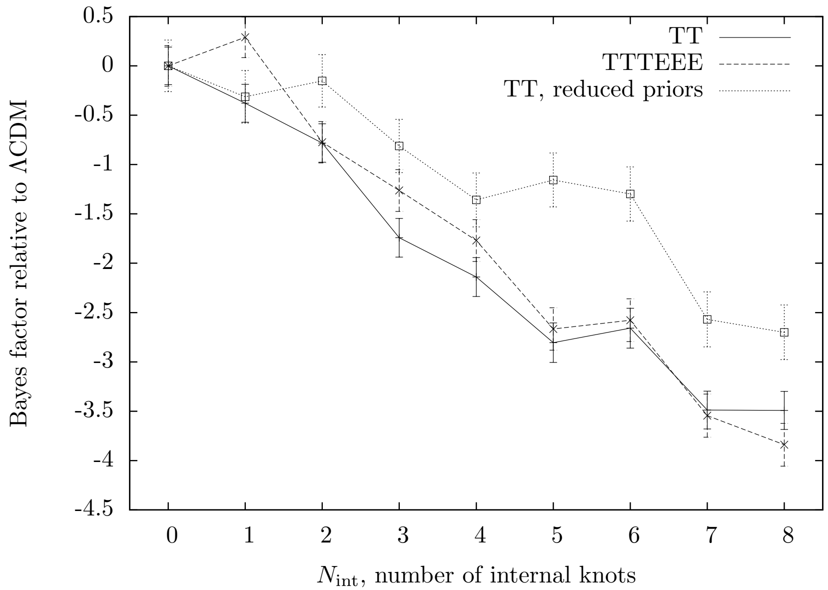
Bayesian reconstruction of the PPS indicate that a power-law is a very good fit in the range $0.005\,\mathrm{Mpc}^{-1} \lesssim k \lesssim 0.2\,\mathrm{Mpc}^{-1}$.

The pattern at smaller k connected with the low-l deficit and the feature at l≈20 is substantially unchanged. Cosmic variance and Planck noise in polarization do not allow to make any definite statement at these scales.

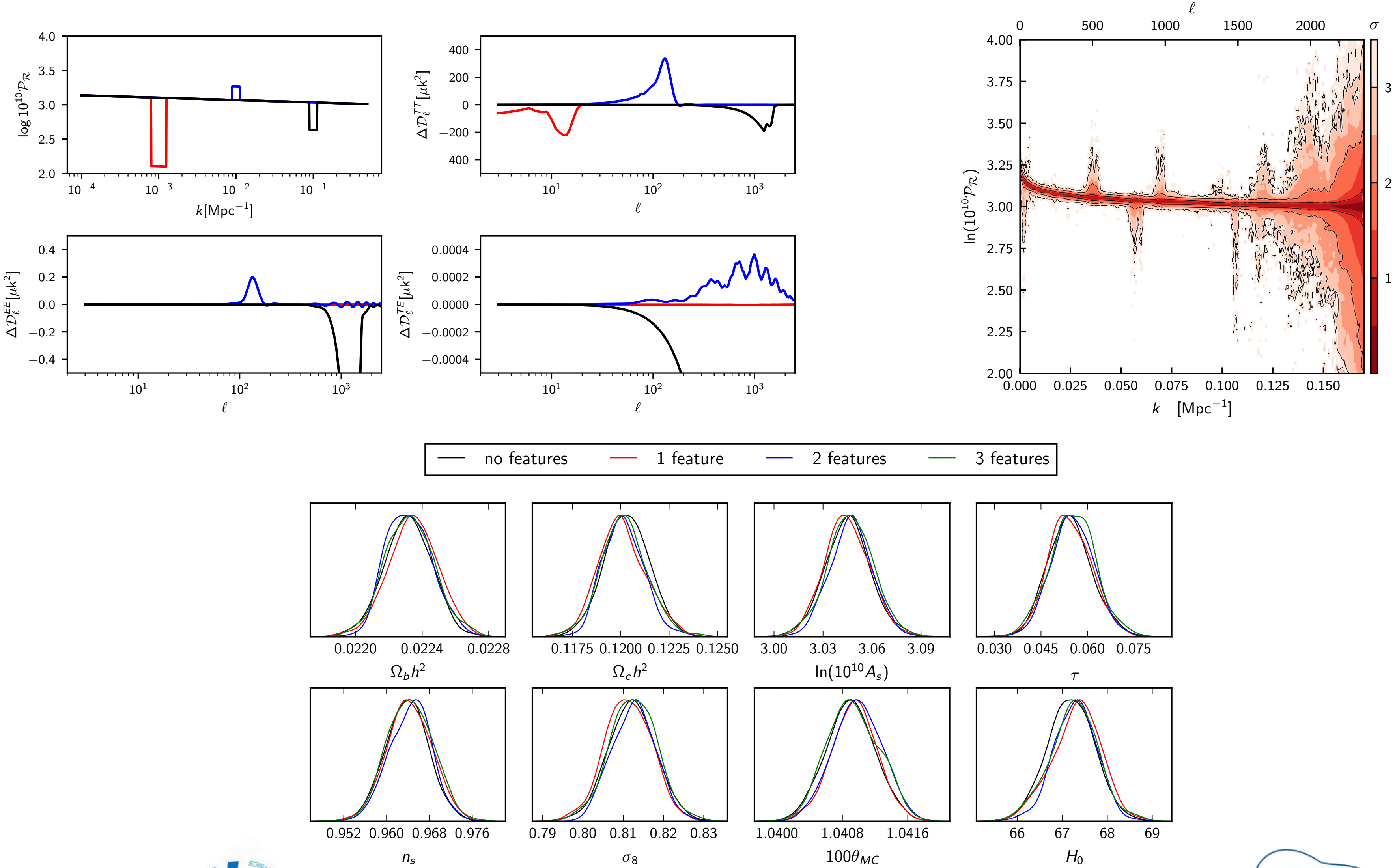




No evidence at statistically significant level over a simple power-law spectrum.

These results are confirmed by the other two independent methods.

What model-independent constraints can be placed on the PPS?



Is there evidence for features in the PPS?

Update of the Bayesian analysis for the search for parametrized features in the Planck power spectra

Dedicated numerical analysis for axion monodromy

Combined power spectrum and bispectrum analysis for oscillatory features

Is there evidence for features in the PPS?

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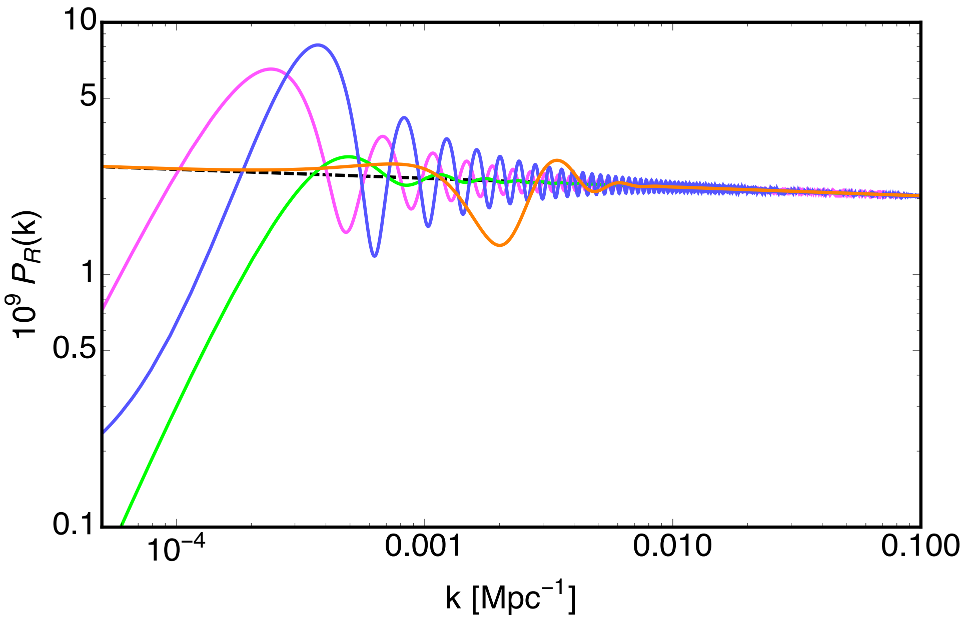
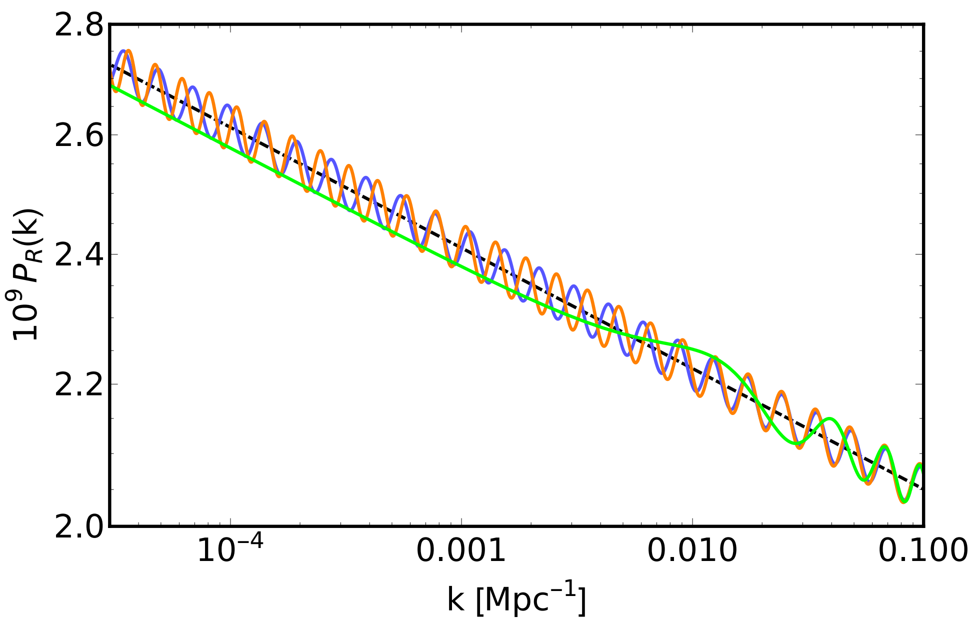
"Local" features



"Oscillatory" features



Potential with a step

Inflation preceded by a kinetic stage

Inflation preceded by radiation stage

Discontinuity in the first derivative of the potential

Logarithmic oscillations

Running logarithmic oscillation

Linear oscillations

Is there evidence for features in the PPS?

Analysis with unbinned TT, EE and joint TT,TE,EE likelihoods by keeping fixed nuisance parameters to their base-LambdaCDM values.

	Step			Kin cutoff			Rad cutoff			Kink cutoff		
	TT	EE	TT,TE,EE	TT	EE	TT,TE,EE	TT	EE	TT,TE,EE	TT	EE	TT,TE,EE
$\Delta\chi^2_{\rm eff}$	−7.0	−5.2	−5.4	−1.2	0.0	−0.9	−0.2	−4.7	−0.0	−2.1	−7.4	−1.1
$\ln B$	0.0	−0.2	0.1	0.0	−0.2	0.0	−0.8	0.0	−0.7	−0.4	0.1	−0.4
$\mathcal{A}_{\rm s}$	0.29	0.19	0.38	...	...	...	...	...	...	...	...	...
$\log_{10}(k_{\rm s})$	−3.11	−3.47	−3.09	...	...	...	...	...	...	...	...	...
$\ln x_{\rm s}$	0.57	2.17	0.15	...	...	...	...	...	...	...	...	...
$\log_{10}\left(k^{\rm c}_{Y}\right)$	...	...	...	−3.70	−4.98	−3.72	−4.87	−3.48	−4.86	−3.05	−3.48	−3.91
$R_{\rm c}$	...	...	...	...	...	...	...	...	...	−0.02	0.33	−0.22

Table 12. Best-fit effective $\Delta\chi^2$ and logarithm of the Bayes factors with respect to a featureless power spectrum, as well as best-fit feature parameters, for the step and cutoff models. Negative values of $\ln B$ indicate a preference for a power-law spectrum, while positive ones prefer the feature model. Wavenumbers are in units of $\rm Mpc^{-1}$.

No preference of these models with respect to the featurless power spectrum for “local” models in temperature, polarization or in their combination.

For the step model the improvement in the fit of these models to the low-l deficit slightly decreases with respect to Planck 2015 because of the lower and more precise value of the optical depth.

Is there evidence for features in the PPS?

	Log osc			Running log osc			Lin osc		
	TT	EE	TT,TE,EE	TT	EE	TT,TE,EE	TT	EE	TT,TE,EE
$\Delta\chi^2_{\rm eff}$	−8.5	−13.5	−11.0	−9.3	−16.5	−11.4	−4.2	−9.0	−10.8
$\ln B$	−1.5	−0.2	−0.9	−1.3	0.2	−0.5	−1.8	−1.3	−0.8
$\mathcal{A}_X$	0.024	0.073	0.014	0.028	0.082	0.016	0.024	0.046	0.015
$\log_{10}\omega_X$	1.51	1.72	1.26	1.50	1.71	1.26	1.74	1.84	1.05
$\varphi_X/(2\pi)$	0.60	0.07	0.07	0.68	0.62	0.11	0.34	0.81	0.56
$\alpha_{\rm rf}$	...	...	...	−0.028	0.022	−0.021	...	...	...

Table 13. Same as Table 12, but for the oscillatory feature models.

– No preference with respect to the featurless power spectrum for “oscillatory” models in temperature, polarization or in their combination. In general the improvement in the fit is similar to Planck 2015.

– It is worth pointing out that models with oscillations linear in k and a frequency of the corresponding modulation of the angular power spectra matching that of the CMB's acoustic oscillations for $\log_{10}\omega_{\rm lin} \simeq 1.15$ mimic the (unphysical) effect of AL. Further analysis shows that explaining the lensing excess would require a model with a tuned scale-dependent linear modulation of the primordial spectrum. Although these two models are not degenerate when taking into accout TT, TE, and EE, Planck TE and EE data are not sensitive enough to make this distinction.

Is there evidence for features in the PPS?

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First joint analysis of power- and bi-spectrum for two “oscillatory” features models with logarithmic and linear oscillations

$$\mathcal{P}_{\rm log}(k) = \mathcal{P}_0(k)\left[1 + A_{\rm log}\cos\left(\omega_{\rm log}\log\frac{k}{\tilde{k}_0} + \phi_{\rm log}\right)\right], \qquad B_{\rm log}(k_1,k_2,k_3) = \frac{B_{\rm log}A_{\rm s}}{k_1^2k_2^2k_3^2}\cos\left(\omega_{\rm log}\log\sum\frac{k_i}{\tilde{k}_0} + \tilde{\phi}_{\rm log}\right).$$

$$\mathcal{P}_{\rm lin}(k) = \mathcal{P}_0(k)\left[1 + A_{\rm lin}\sin\left(2\omega_{\rm lin}\frac{k}{\tilde{k}_0} + \phi_{\rm lin}\right)\right] \qquad B_{\rm lin}(k_1,k_2,k_3) = \frac{B_{\rm lin}A_{\rm s}}{k_1^2k_2^2k_3^2}\cos\left[\omega_{\rm lin}\left(\sum\frac{k_i}{\tilde{k}_0}\right) + \tilde{\phi}_{\rm lin}\right].$$

Build Likelihood for the bispectrum using the 2015 posterior of the modal estimator for log and linear feature bispectra

Add as external likelihood to Planck TT,TE,EE (unbinned) lowE + lensing

Using polychord sampler, varying all cosmo pars and nuisance pars

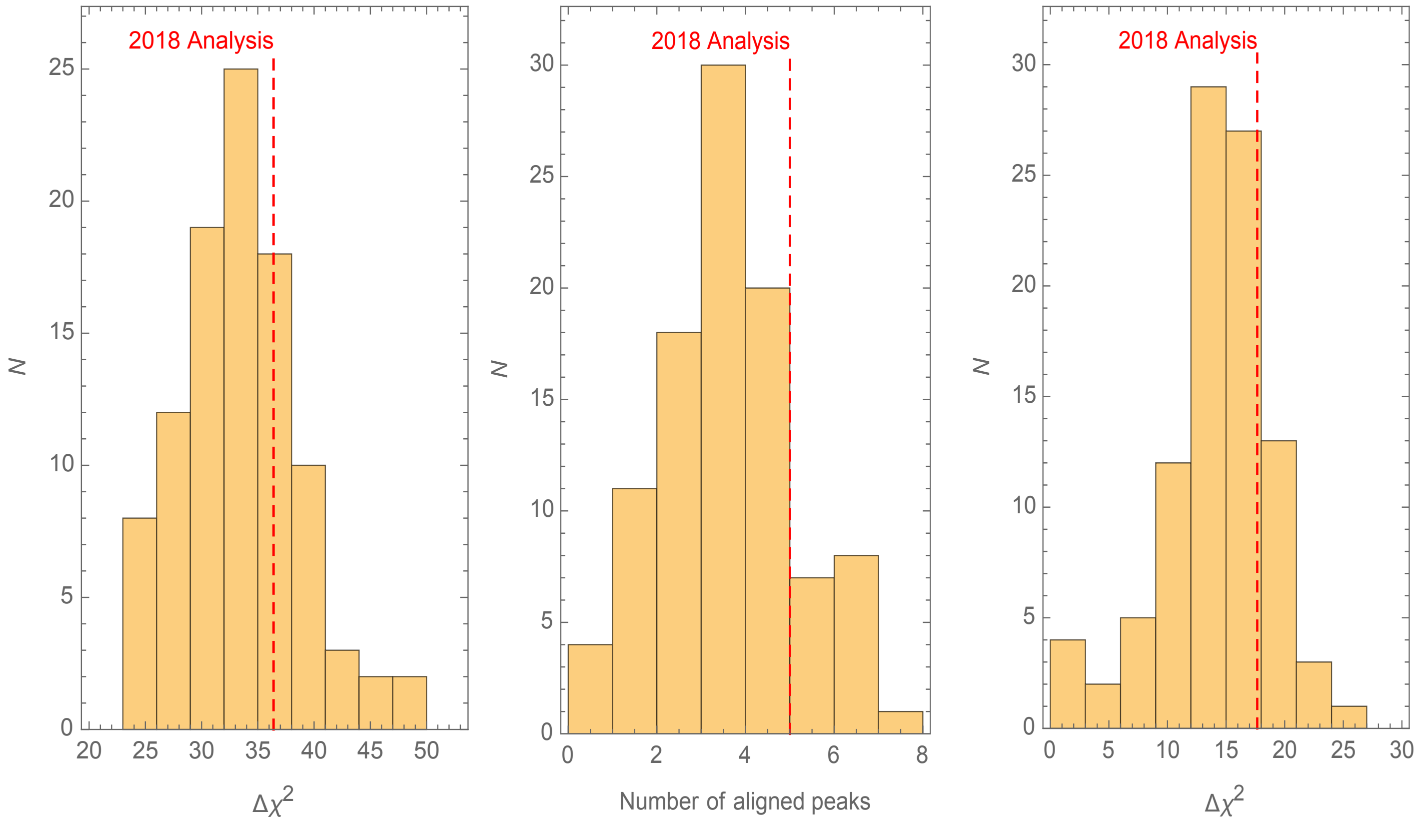
Address significance of fits combined with bispectrum

Simulate noisy mock bispectra, drawn from covariance

Address following questions:

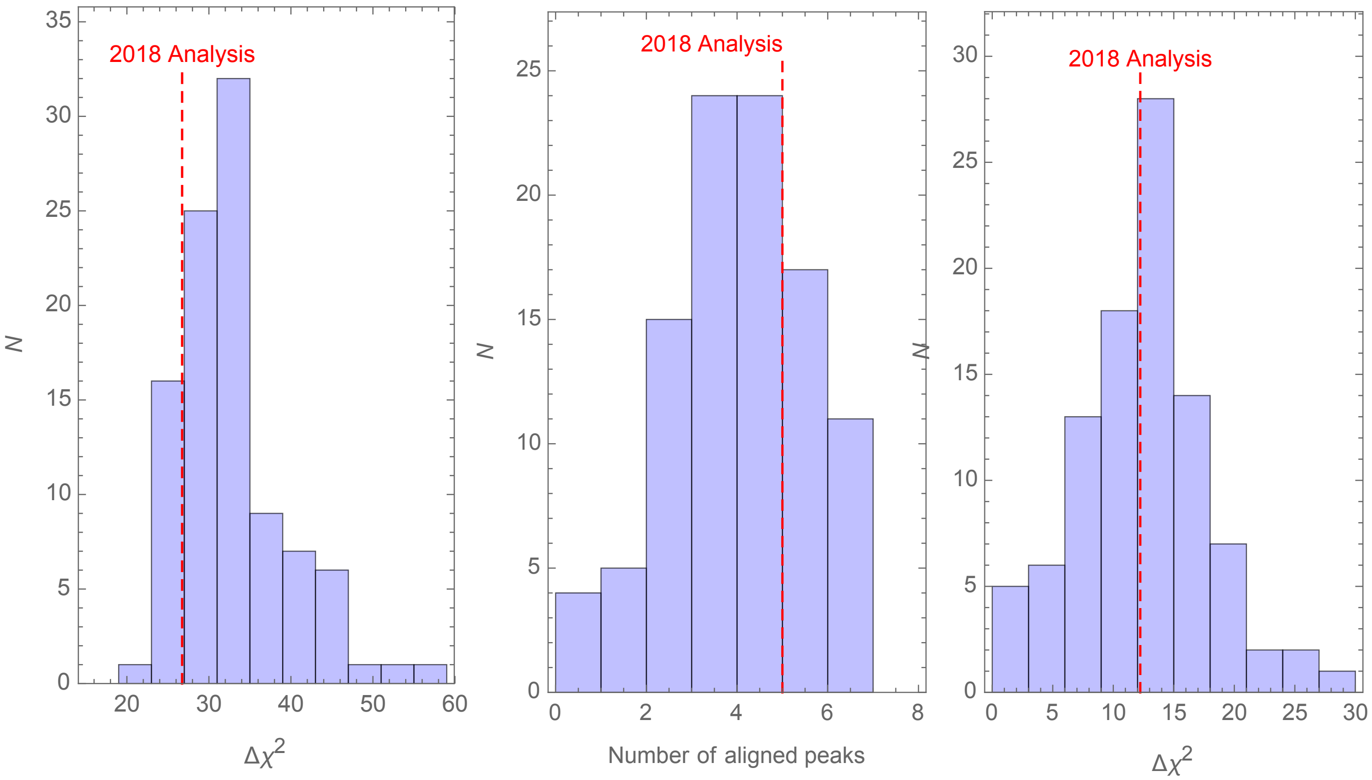
- Improvement of fit
- Number of aligned peaks
- Mean improvement of fit of aligned peaks

Is there evidence for features in the PPS?



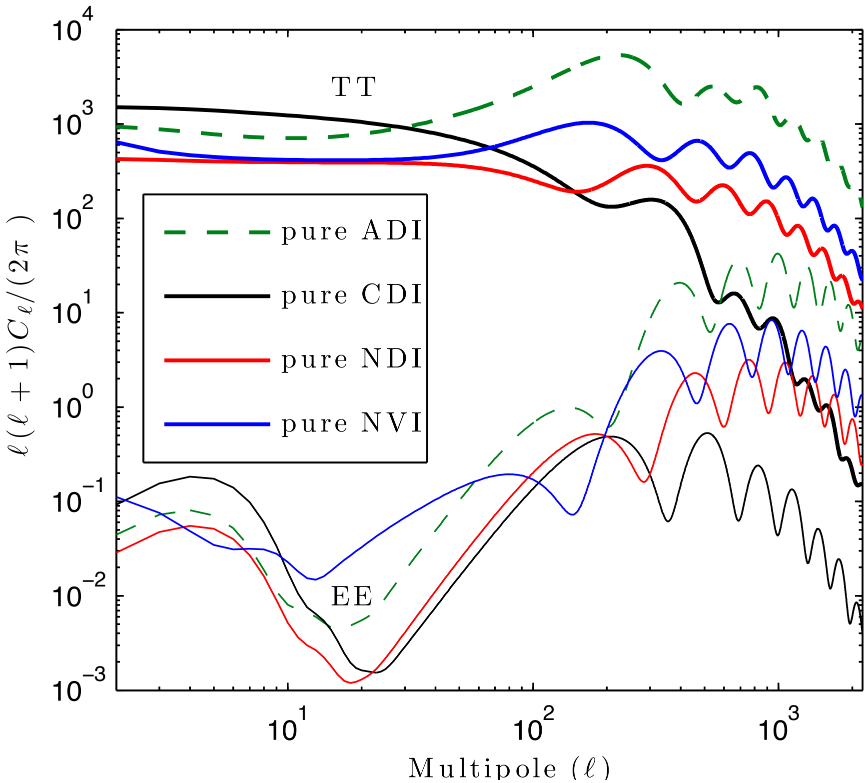
No evidence for correlated logarithmic features

Is there evidence for features in the PPS?



No evidence for correlated linear features

Were the primordial cosmological perturbations solely adiabatic?



$$\mathcal{R} \approx C(k\tau)^\alpha \quad \text{for} \quad k\tau << 1$$

$$\alpha = 0 \quad \text{Curvature}$$

$$\alpha = 1 \quad C \approx \Omega_c(\Omega_b) \quad \text{CDI(BI)}$$

$$\alpha = 1 \quad \text{NVI}$$

$$\alpha = 2 \quad \text{NDI}$$

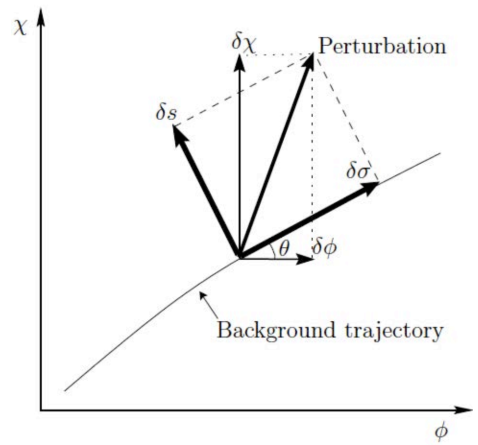
... i.e. was inflation driven by more than one field?

Single field inflation cannot generate isocurvature fluctuations

Isocurvature perturbations can be generated in multi-field inflationary models:

$$\mathcal{R} = -H \frac{\sum_{i=1}^{N} \dot{\phi}_i Q_i}{\dot{\sigma}^2}$$

$$\delta s_{ij} = \frac{\dot{\phi}_i Q_j - \dot{\phi}_j Q_i}{\dot{\sigma}} \qquad \dot{\sigma}^2 \equiv \sum_{i=1}^{N} \dot{\phi}_i^2$$



Curvature and isocurvature fluctuations are generated during inflation with non-zero correlation if the trajectory in the field space is curved. Correlation can be generated also during the post-inflationary era.

If the inflaton and the second field decay in two different species then isocurvature perturbations could survive until after nucleosynthesis.

Were the primordial cosmological perturbations solely adiabatic?

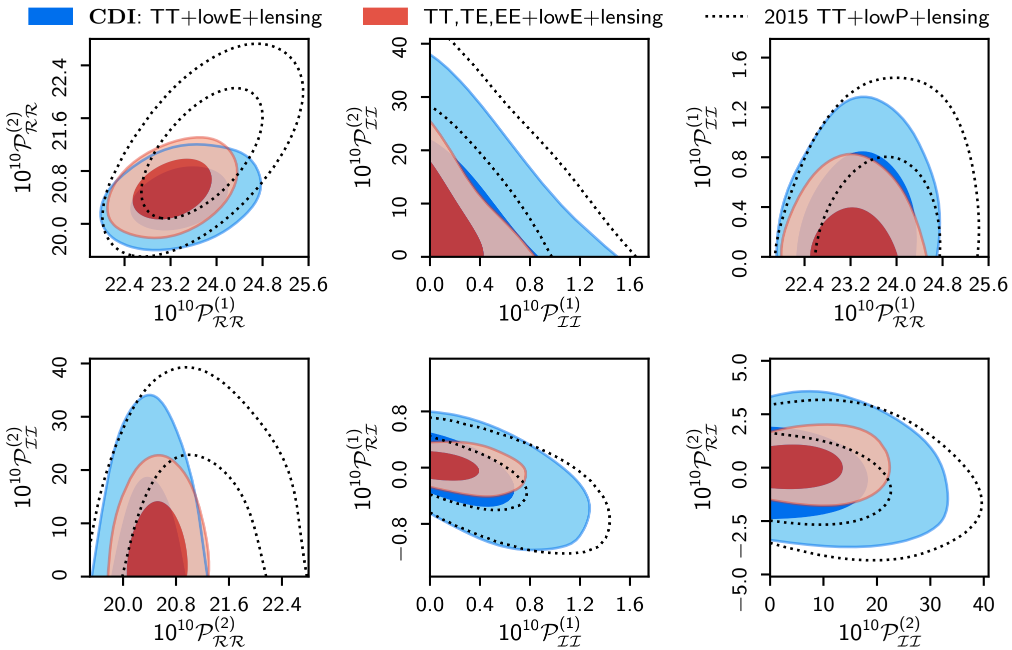
As in PCI15, we adopt a two-scale parametrization at k_1 =0.002 Mpc^{-1} and k_2 =0.1 Mpc^{-1} :

$$\mathcal{P}_{\rm ab}(k) = \exp\left[\left(\frac{\ln(k)-\ln(k_2)}{\ln(k_1)-\ln(k_2)}\right)\ln(\mathcal{P}^{(1)}_{\rm ab}) + \left(\frac{\ln(k)-\ln(k_1)}{\ln(k_2)-\ln(k_1)}\right)\ln(\mathcal{P}^{(2)}_{\rm ab})\right]$$

$$\mathrm{a,b} = \mathcal{R},\mathcal{I}$$

$$\mathcal{I} = \mathcal{I}_{\rm CDI}\,,\mathcal{I}_{\rm NDI}\,,\mathcal{I}_{\rm NVI}$$

and we restrict to scale-independent correlation, allowing for a general correlated mixture of adiabatic and isocurvature with a maximum of three extra parameters.



Analogous quantitative improvements hold for NDI and NVI

Were the primordial cosmological perturbations solely adiabatic?

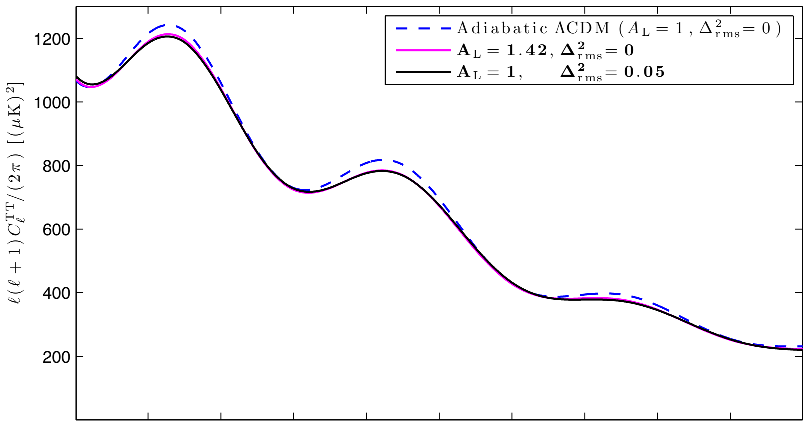
Compensated isocurvature perturbation (CIP) between CDM and baryon isocurvature modes

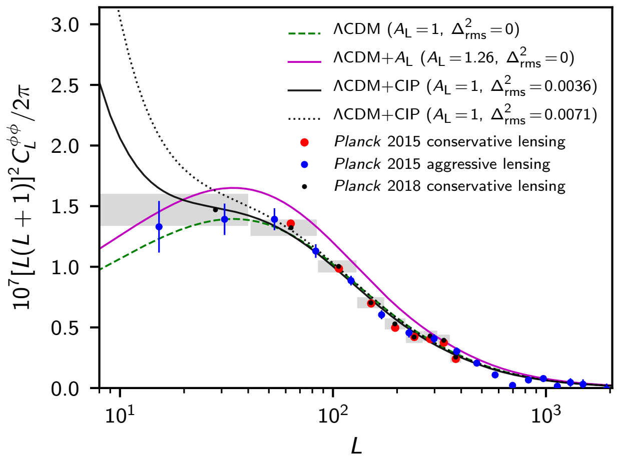
- CIP does not leave any effect at linear order on CMB and matter power spectra (although it modifies the CMB trispectrum)

- CIP can be therefore described at higher order as a large scale modulation of baryon and CDM density whose effect on CMB anisotropies can be described as:

$$C_\ell^{\rm obs}(\bar\Omega_{\rm b},\bar\Omega_{\rm c},\tau,H_0,n_{\rm S},A_{\rm S}) = \frac{1}{\sqrt{2\pi\Delta^2_{\rm rms}}}\int C_\ell\left(\Omega_{\rm b}(\Delta),\Omega_{\rm c}(\Delta),\tau,H_0,n_{\rm S},A_{\rm S}\right)e^{-\Delta^2/(2\Delta^2_{\rm rms})}d\Delta$$

$$\Omega_{\rm b}(\Delta) = (1+\Delta)\bar\Omega_{\rm b} \qquad \Omega_{\rm c}(\Delta) = \bar\Omega_{\rm c} - \bar\Omega_{\rm b}\Delta$$

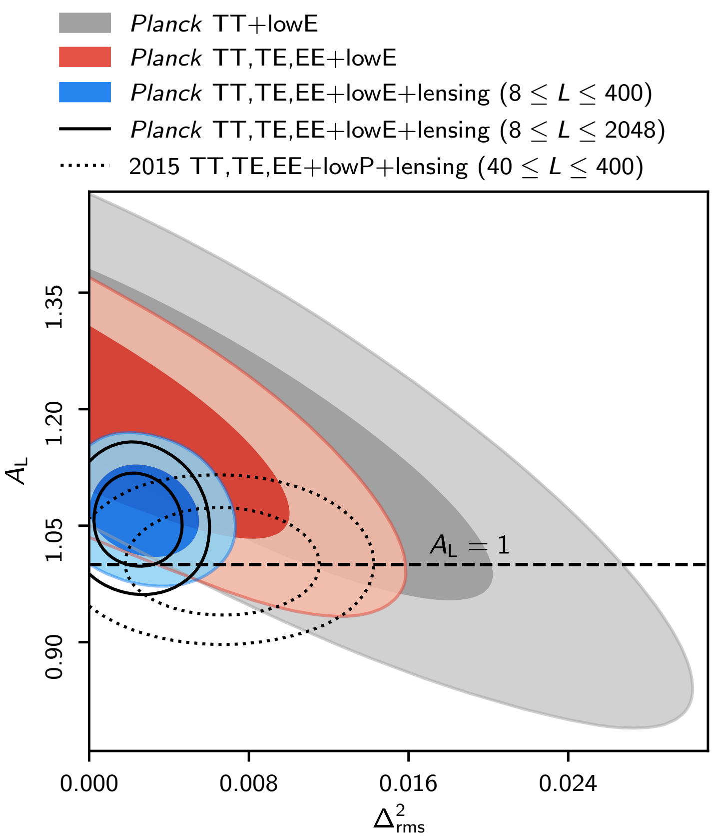




Very similar effect to AL in TE, EE as well.

CIP does not modify the lensing potential at high L, but only at L < 40.

Were the primordial cosmological perturbations solely adiabatic?



Planck data can be also fit by CIP with AL = 1

Planck TT,TE,EE+lowE+lensing result for ΛCDM+CIP (AL = 1):

$$\Delta^2_{\rm rms} = 0.0037^{+0.0016}_{-0.0021} \qquad (68\%\mathrm{CL})$$

Conclusions

Planck 2018 results for inflation are consistent with those from 2013 and 2015 data releases, and most of the conclusions have been strengthened thanks to the improvement in the characterization of polarization at all multipoles.

The key predictions of the simplest inflationary models, i.e. standard single field slow-roll models, provide a good fit to Planck 2018 data.

Due to cosmic variance and/or Planck noise in polarization we did not find compelling theoretical interpretations in terms of new physics beyond the simplest model of inflation for anomalies at low and high multipoles.

Forthcoming E-mode polarization data will be decisive for determining whether the intriguing features in the temperature power spectrum, such as the deficit at $l \approx 20$, the smaller average amplitude at $l \leqq 40$, and other anomalies at higher multipoles require new physics or whether these features are simply statistical fluctuations.

Improved measurements of the B modes promise to constrain inflation even more tightly. Either a detection or an upper limit on r would substantially advance our understanding of inflation and the constraints on the physics of the early Universe.

What is the value of the scalar tilt n_s ?

$$n_\mathrm{s} = 0.9649 \pm 0.0042 \quad (68\%\mathrm{CL})$$

Does n_s depend on the wavelength?

No evidence of running (small 3rd derivative of V(phi))

$$dn_\mathrm{s}/d\ln k = -0.0045 \pm 0.0067 \quad (68\%\mathrm{CL})$$

No evidence of running of running

(small 4th derivative of V(phi))

$$dn_\mathrm{s}/d\ln k = 0.002 \pm 0.010$$
$$d^2 n_\mathrm{s}/d\ln k^2 = 0.010 \pm 0.013 \quad (68\%\mathrm{CL})$$

Is the Universe flat?

$$\Omega_K = 0.0007 \pm 0.0037 \;\; (\mathrm{incl.\,BAO}, 95\%\mathrm{CL})$$

Are tensor modes required?

Small relative amount of gravitational waves (+ BK14)

$$r_{0.002} < 0.064 \quad (95\%\mathrm{CL})$$

A joint fit of (r,nt) in agreement with the slow-roll consistency condition for the tensor tilt (+ BK14 + LIGO/VIRGO)

$$r_{0.002} < 0.069 \quad r_{0.01} < 0.080 \quad (95\%\mathrm{CL})$$
$$r_{0.02} < 0.099 \quad -0.62 < n_\mathrm{t} < 0.53$$

Which inflationary models are best able to account for the data?

Within a representative selection of slow-roll inflationary models, R2, D-brane inflation, alpha attractors, ... provide a similar fit to Planck and BK14 data by taking into account the uncertainties in reheating.

In combination with BK14, Planck data does not support any evidence of inflationary dynamics beyond slow-roll.

What model-independent constraints can be placed on the primordial power spectrum?

Three different methodologies for the reconstruction of the PPS indicate that a power-law is a very good fit in the range $0.005\,\mathrm{Mpc}^{-1} \lesssim k \lesssim 0.2\,\mathrm{Mpc}^{-1}$. Cosmic variance and Planck noise in polarization do not allow to make any definite statement on anomalies on smaller Fourier scales.

Is there evidence for features in the primordial power spectrum?

No evidence at a statistically significant level for features in the PPS from Planck power spectra alone. This result is further confirmed by a joint power spectrum-bispectrum analysis for models with superimposed linear or logarithmic oscillations.

Were the primordial cosmological perturbations solely adiabatic?

Yes, Planck provide a very stringent test for adiabatic initial conditions. No evidence for multi-field inflation.

$$\beta_\mathrm{iso}^\mathrm{axion} < 0.038 \quad (95\%\mathrm{CL})$$

Were the primordial fluctuations statistically isotropic?

No evidence for physical models producing a dipolar or quadrupolar modulation superimposed to a smooth primordial power spectrum.

Additional slides

Are tensor modes required?

As in PCI15, we adopt a two-scale parametrization with primary parameters the tensor-to-scalar ratios at $k_1 = 0.002\ \mathrm{Mpc}^{-1}$ and $k_2 = 0.02\ \mathrm{Mpc}^{-1}$ in order to analyze data when the theoretical prior nt = −r/8 is relaxed.

Planck 2018 + BK14

$$r_{0.002} < 0.044 \qquad r_{0.01} < 0.091$$
$$r_{0.02} < 0.232 \qquad -0.34 < n_t < 2.63 \qquad (95\%\mathrm{CL})$$

A stochastic background of gravitational waves (GW) with a blue tensor tilt can be further constrained at much shorter wavelength as those probed by ground-based interferometers dedicated to the direct detection of GWs. The LIGO/VIRGO upper bound (Abbott et al. 2016) on the GW energy density translates in an upper bound on r on short scales.

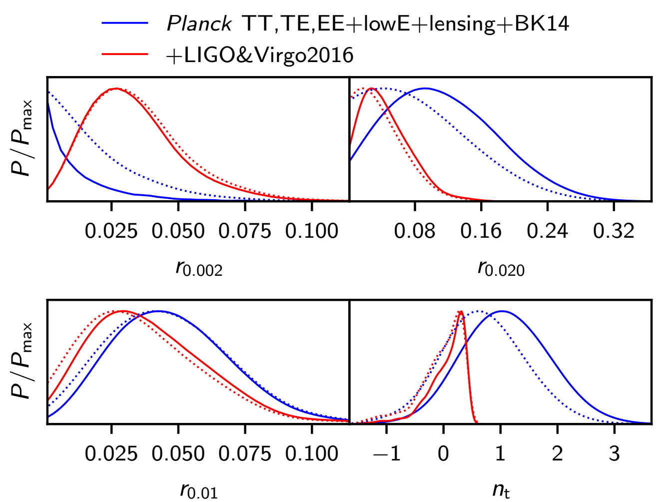
$$\Omega_{\mathrm{GW}}(f) \leq 1.7 \times 10^{-7} \quad (95\%\mathrm{CL}) \quad \text{at } f = 20\,\mathrm{Hz}$$

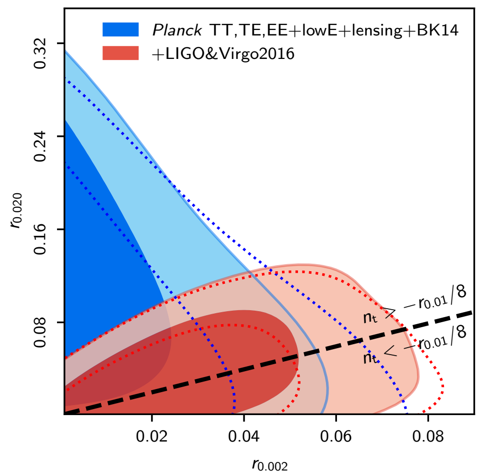
$$\Omega_{\mathrm{GW}}(k) = \frac{k}{\rho_{\mathrm{critical}}} \frac{d\rho_{\mathrm{GW}}}{dk} = \frac{A_{t1}(k/k_1)^{n_t}}{24 z_{\mathrm{eq}}}$$

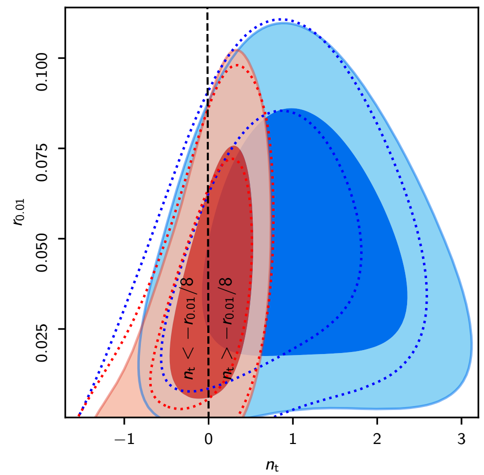
$$r \leq 2.6 \times 10^{7} \quad (95\%\mathrm{CL}) \quad \text{at } k = 1.3 \times 10^{16}\,\mathrm{Mpc}^{-1}$$

Planck 2018 + BK14 + LIGO/VIRGO

$$r_{0.002} < 0.069 \qquad r_{0.01} < 0.080$$
$$r_{0.02} < 0.099 \qquad -0.62 < n_t < 0.53 \qquad (95\%\mathrm{CL})$$



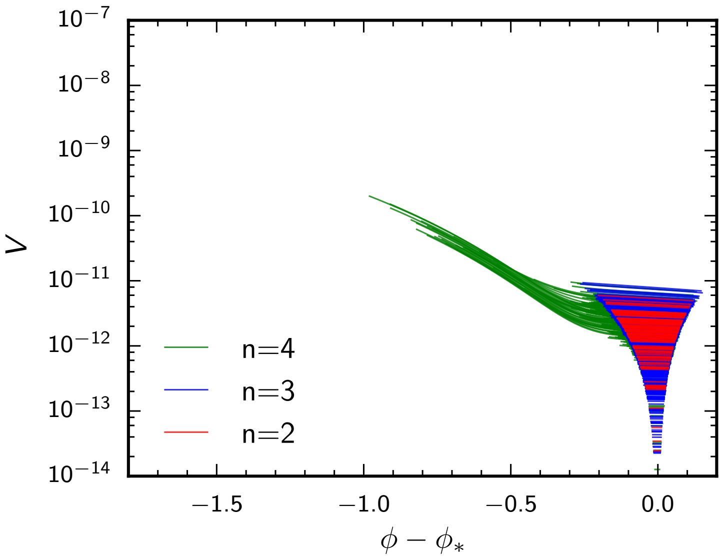


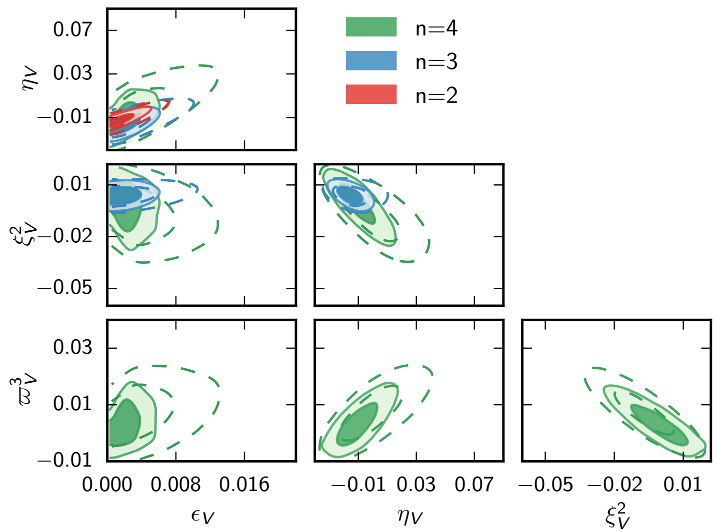


Which inflationary models are best able to account the data?

planck

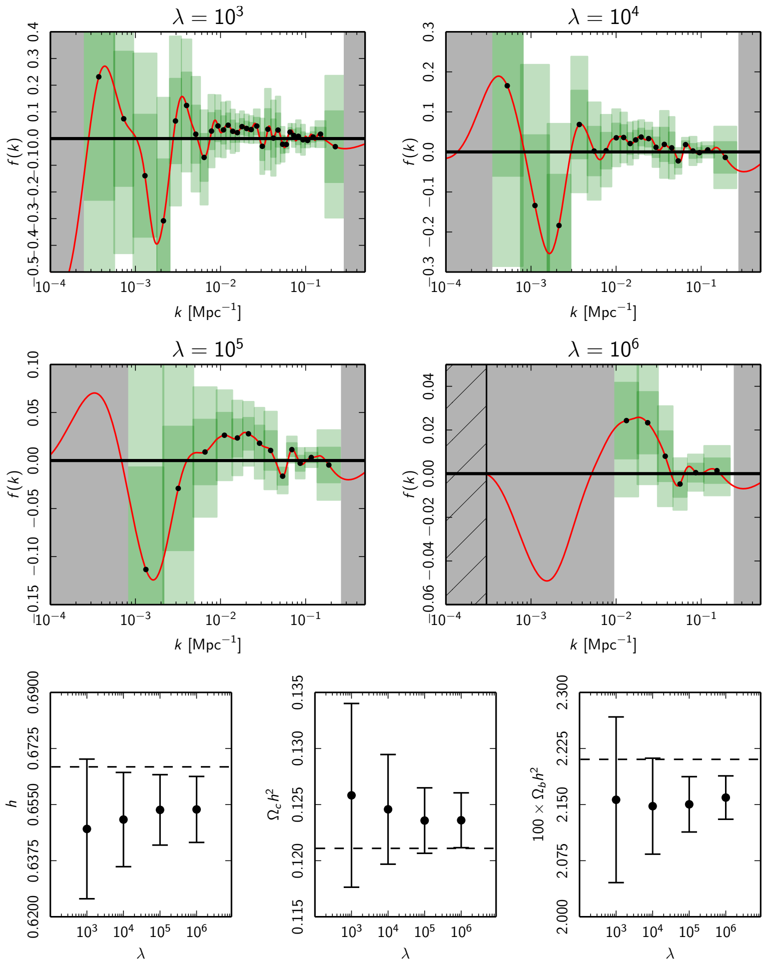
Reconstruction of the inflationary potential as a Taylor expansion without any assumption about slow-roll or about the end of inflation leads to no evidence for large third of fourth derivatives of the potential, i.e. no evidence of physics beyond slow-roll.

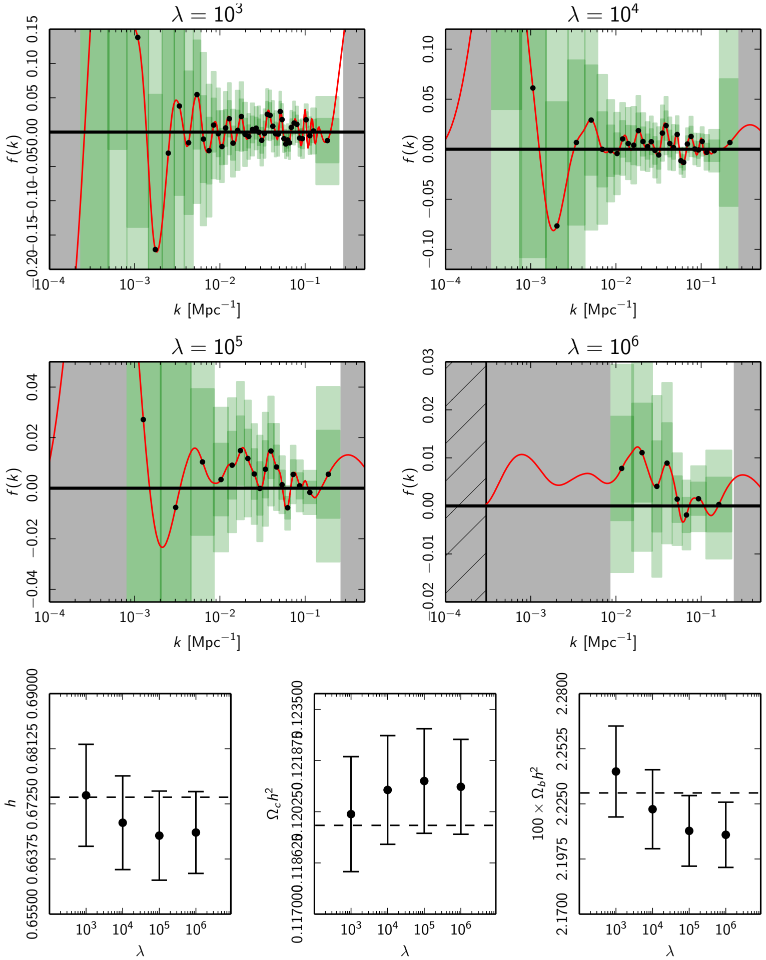




Planck TT,TE,EE + lowE + BAO

What model-independent constraints can be placed on the PPS?





Were the primordial fluctuations statistically isotropic?

planck

$$\mathcal{P}_{\mathcal{R}}(\mathbf{k}) = \mathcal{P}^0_{\mathcal{R}}(k)\left[1 + g(k)\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{d}}\right)^2\right] = \mathcal{P}^0_{\mathcal{R}}(k)\left[1 + \frac{1}{3}g(k) + \sum_m g_{2m}(k)\, Y_{2m}(\hat{\mathbf{k}})\right]$$

$$g_{2m}(k) \equiv \frac{8\pi}{15} g(k)\, Y^*_{2m}(\vec{\hat{d}})$$

$$g(k) = g_*(k/k_*)^q$$