Review 3

1. Evaluate line integrals:
   (a) $\int_C x^2 y\,ds$, where C is parameterized by $\bar{r}(t) = (\cos t)\bar{i} + (\sin t)\bar{j} + t\bar{k}$, $0 \le t \le \frac{\pi}{2}$.
   (b) $\int_C x^3\,ds$, where C is the portion of the unit circle in the first quadrant oriented counterclockwise.
   (c) $\int_C (x\,dx + y\,dy + z\,dz)$, where C is the line segment from $(0,1,0)$ to $(1,2,3)$.
   (d) $\int_C (y\,dx - x\,dy)$, where C is parameterized by $\bar{r}(t) = (\ln t)\bar{i} + t\bar{j}$, $1 \le t \le 2$.
2. Find the work done by the force
$$\bar{F} = \frac{-x\bar{i} - y\bar{j} - z\bar{k}}{(x^2 + y^2 + z^2)^{3/2}}$$
   when the object moves the point $(1,0,0)$ to the point $(1,1,1)$.
3. Using Green's theorem, compute line integrals:
   (a) $\int_C (ye^x\,dx + \cos(x^2)\,dy)$, where C is the square with vertices $(\pm 1, \pm 1)$ oriented counterclockwise.
   (b) $\int_C (2y\,dx + x\,dy)$, where C is the leaf of $r = \cos 3\theta$, $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$, oriented clockwise.
4. Using line integrals, compute the area of the region bounded by $\bar{r}(t) = (t^3 + t + 1)\bar{i} + (1 - t^2)\bar{j}$, $-1 \le t \le 1$, above and by the x-axis below.
5. Compute surface integrals:
   (a) $\iint_\Sigma \frac{xy}{x^2+y^2}\,dS$, where Σ is the part of the paraboloid $z = x^2 + y^2$ below the plane $z = x$.
   (b) $\iint_\Sigma (x + y)\,dS$, where Σ is the triangle with vertices $(0,0,1)$, $(2,0,0)$, $(0,1,0)$.
6. Evaluate $\iint_\Sigma (\bar{F} \cdot \bar{n})\,dS$ where
   (a) $\bar{F} = x\bar{i} + y\bar{j} + z\bar{k}$; Σ is the part of the surface $z = x^2 + y^2$ inside of the cylinder $x^2 + y^2 = 1$; $\bar{n}$ is directed upward.
   (b) $\bar{F} = \bar{i} + z\bar{j} + x\bar{k}$; Σ is the portion of the cylinder $x^2 + z^2 = 1$ with $0 \le y \le 1$; $\bar{n}$ is directed outward.
7. Compute $\int_C (z^2\,dx + x^2\,dy + y^2\,dz)$, where C is the intersection of the sphere $x^2 + y^2 + z^2 = 2$ and the plane $z = 1$, and C has counterclockwise orientation as viewed from above.
8. Compute $\int_C ((y + 2z)\,dx + (3x + z)\,dy + (x + y)\,dz)$, where C is the intersection of the surfaces $z = x^2 + 2y^2$ and $z = 1 - 2y^2$, and C has clockwise orientation as viewed from above.
9. Compute $\iint_\Sigma (\bar{F} \cdot \bar{n})\,dS$ where $\bar{F} = xy\bar{i} + z\bar{j}$, Σ is the boundary of the portion of the ball $x^2 + y^2 + z^2 \le 1$ in the first octant, and $\bar{n}$ is directed outward.
10. Compute $\iint_\Sigma (\bar{F} \cdot \bar{n})\,dS$ where $\bar{F} = x^2\bar{i} + y\bar{j} + z\bar{k}$, Σ is the boundary of the solid region which is bounded by $z = x^2 + y^2$ below and by $z = 2 - x^2 - y^2$ above, and $\bar{n}$ is directed outward.