On behavioral complementarity and its implications

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Complementary goods

Complementary goods

Coffee and Sugar
Tea and Lemon
Peanut butter and Jelly
Cheese and Wine
Beer and Pretzels
Gin and Tonic

Matters for IO models and practical problems.
(e.g. firms' pricing policy).

Consumer Theory

Explain consumer behavior: choices of *consumption bundles* $x \in X \subseteq \mathbb{R}^n$

Consumer Theory

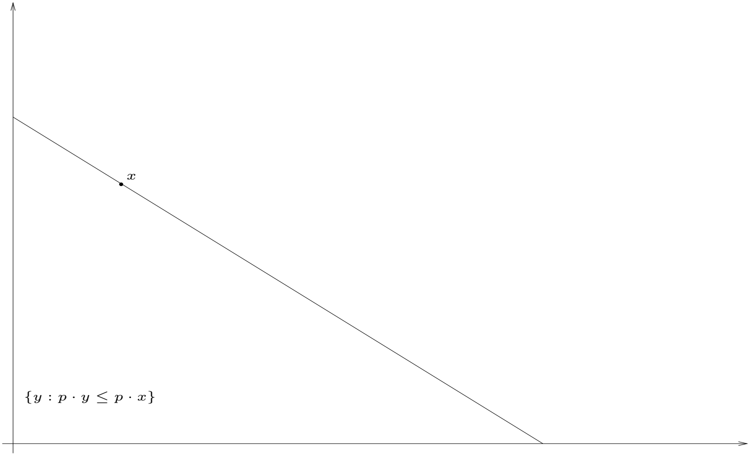
Postulate a *utility function*

$$u : X \rightarrow \mathbb{R}$$

Consumer behaves as if

$$\begin{aligned} \max \;\; & u(x) \\ & x \in \{y \in X : p \cdot y \leq m\} \end{aligned}$$

Consumer Theory



Consumer Theory

Problem: we don't observe u; it's a theoretical construct.

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Consider data $(x^k, p^k), k = 1, \ldots, K$ on prices and consumption.
Say that u *rationalizes* the data if, for all k,

$$y \neq x^k \text{ and } p^k \cdot y \leq p^k \cdot x^k \Rightarrow u(y) < u(x^k)$$

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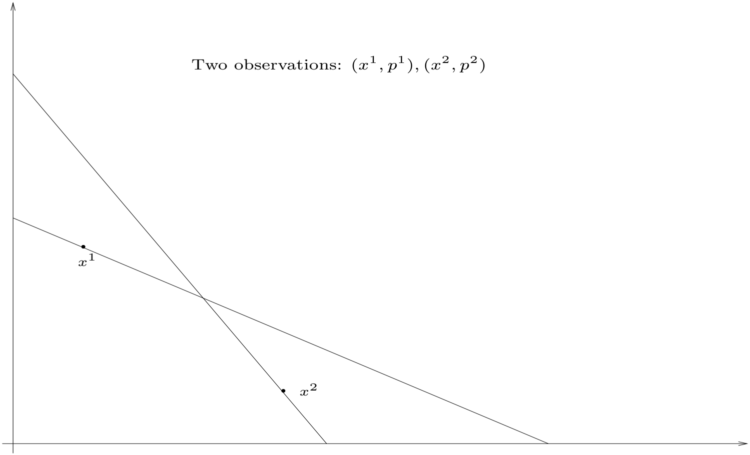
Can we rationalize *anything* choosing the appropriate u ?


Two observations: $(x^1, p^1), (x^2, p^2)$
x^1
x^2

Afriat (1967)

Consider data $(x^k, p^k), k = 1, \ldots, K$.

Theorem

The following are equivalent

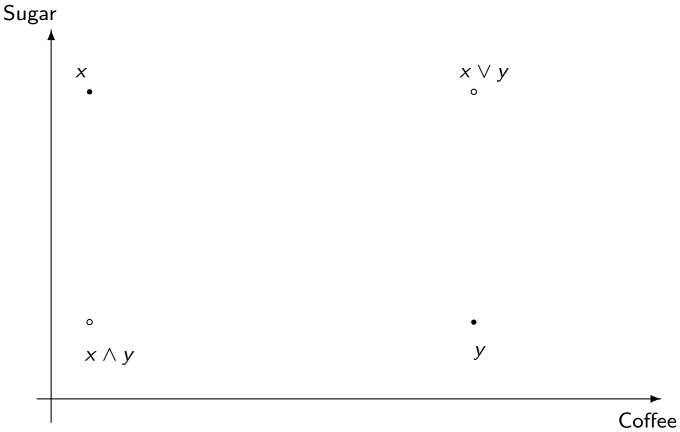
1. *The data can be rationalized.*
2. *The data satisfy GARP.*
3. *The data can be rationalized by a weakly monotonic, continuous, and concave utility.*

Back to complementary goods

How do choices between
Coffee and Sugar
and between
Coffee and Tea
differ ?
(choices $\longrightarrow$ testing)

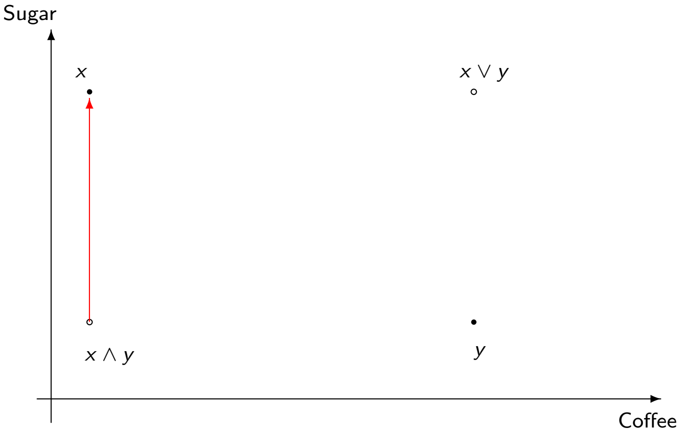
How do preferences differ ?
(preferences $\longrightarrow$ modeling)

Complementarity in utility (Edgeworth-Pareto)



$$u(x) - u(x \wedge y) \leq u(x \vee y) - u(y)$$

Complementarity in utility (Edgeworth-Pareto)



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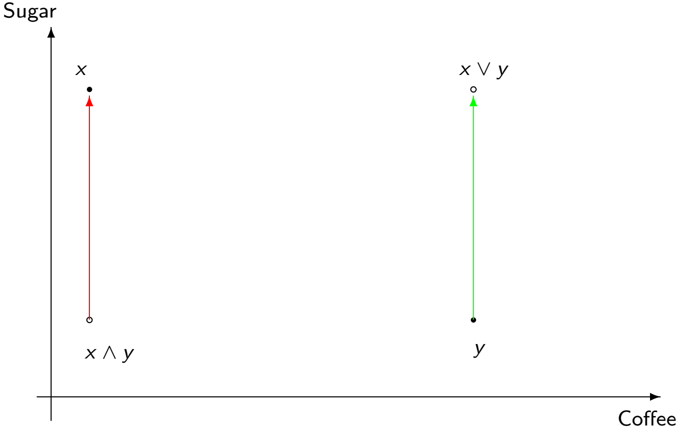

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Theorem (Chambers & Echenique (2006))

Data, $(x^k, p^k), k = 1, \ldots, K$, can be rationalized if and only if it can be rationalized by a supermodular utility

Behavioral complementarity

The standard notion of complementarities based on utility has no implication for (observable) choices.

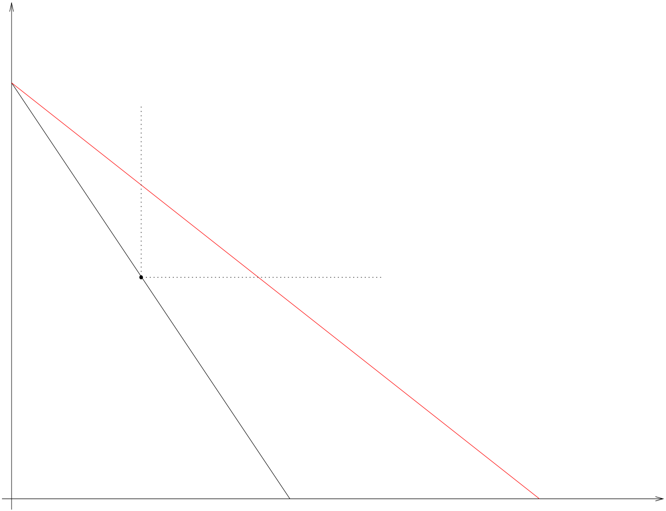
So we study a *behavioral* notion.

Behavioral complementarity

$\downarrow$ price of coffee $\Rightarrow$ $\uparrow$ demand for sugar.

"Behavioral" $\neq$ condition on preferences.

Behavioral complementarity



Behavioral complementarity

Essentially a property of pairs of goods.

Ex: Coffee, Tea and Sugar.
Use more sugar for tea than for coffee.

What if have n goods ?

Standard practice: estimate cross elasticity assuming demand functional form.

We need separability in preferences (and possibly aggregation).

e.g. heating and housing.

Demand

A *demand* is a function $D : \mathbb{R}^2_{++} \times \mathbb{R}_+ \to \mathbb{R}^2_2$ s.t.

- $p \cdot D(p, I) = I$ (Walras Law).
- $\forall t > 0$, $D(tp, tI) = D(p, I)$ (Homogeneity of degree zero)

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i.e. $\exists$ w. mon. u with

$$D(p, I) = \text{argmax}_{\{x : p \cdot x \leq I\}} u(x)$$

Behavioral complementarity – Two Models

- Demand: $D(p, I)$ (Nominal Income).
  D satisfies *complementarity* if
  $p \leq p' \Rightarrow D(p', I) \leq D(p, I)$.
- Demand: $D(p, p \cdot \omega)$ (Endowment Income)
  D satisfies *(weak) complementarity* if, for every p, ω there is a p' such that

$$\left[D_1(p', p' \cdot w) - D_1(p, p \cdot w)\right]\left[D_2(p', p' \cdot w) - D_2(p, p \cdot w)\right] \geq 0$$

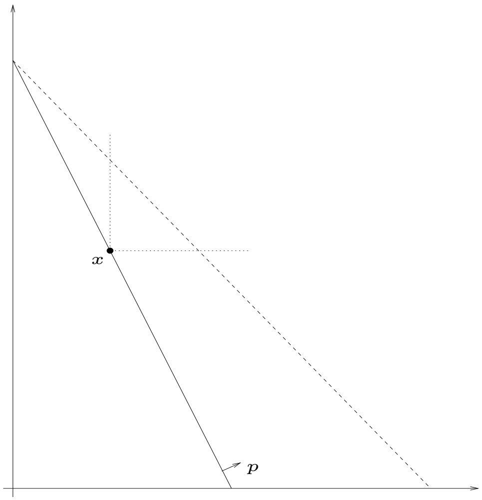
Results (vaguely)

- Necessary and sufficient condition for observed demand to be consistent with complementarity (testable implications).
- Necessary and sufficient condition (within domains) for preferences to generate complements in demand.
- Differences in Nom. Income vs. Endowment Income.

Nominal Income – Testable Implications

Expenditure data: (x, p) (x', p')
(Samuelson (1947), Afriat (1967), etc.)

Nominal Income – Testable Implications



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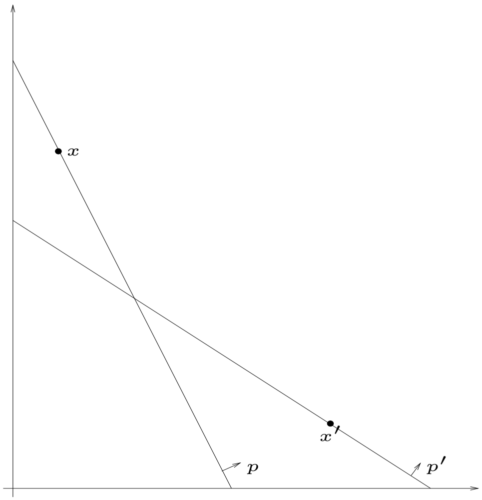

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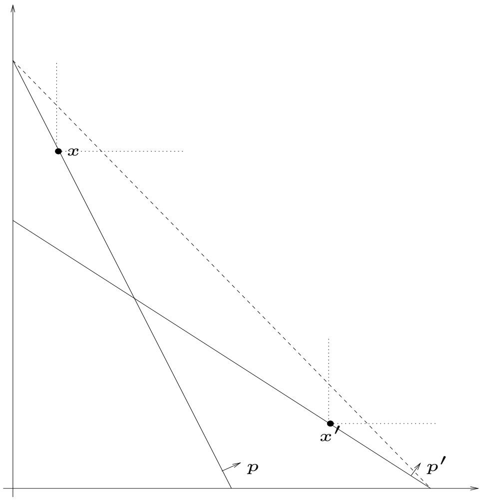
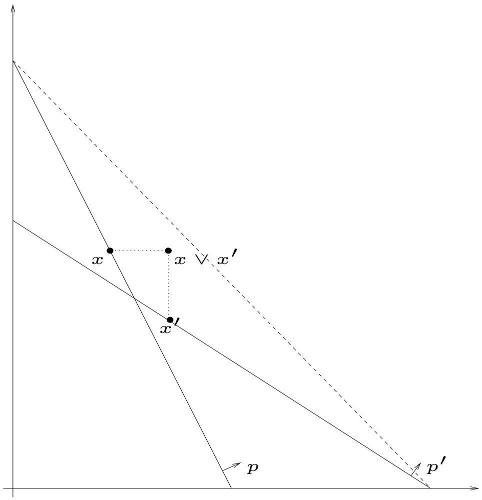


Nominal Income – Testable Implications

So $x \vee x' \notin B \vee B'$

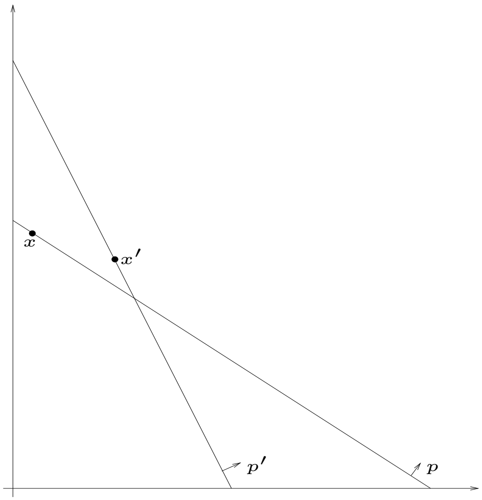
Nominal Income – Testable Implications



Nominal Income – Testable Implications

Necessary condition 1 : $x \vee x' \in B \vee B'$

Nominal Income – Testable Implications



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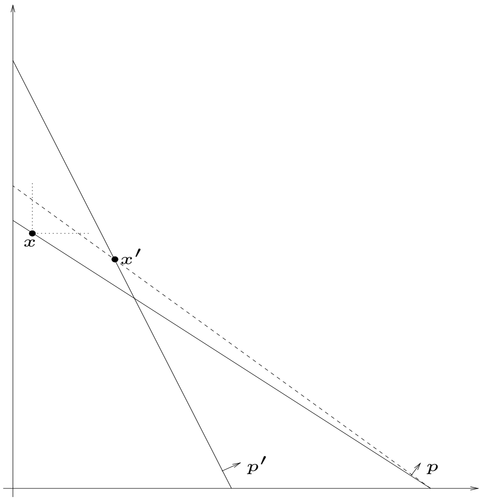

Nominal Income – Testable Implications

Necessary condition 2 : a strengthening of WARP.

Nominal Income

By homogeneity, $D(p/I, 1) = D(p, I)$.
So fix income $I = 1$ and write $D(p)$.
Note: $B \vee B'$ is budget with $p \wedge p'$ and $I = 1$.

Nominal Income

An *observed demand function* is a function $D : P \to \mathbb{R}^2_+$

- $P \subseteq \mathbb{R}^2_{++}$ is finite
- $p \cdot D(p) = 1$

Nominal Income

Let $D : P \to \mathbb{R}^2_+$ be an observed demand.

Theorem (Observable Demand)

D is the restriction to P of a rational demand that satisfies complementarity iff $\forall p, p' \in P$

1. $(p \wedge p') \cdot (D(p) \vee D(p')) \leq 1$.
2. *If* $p' \cdot D(p) \leq 1$ *and* $p'_i > p_i$ *then* $D(p')_j \geq D(p)_j$ *for* $j \neq i$.

Nominal Income

Theorem (Continuity)

Let $D : \mathbb{R}^2_{++} \to \mathbb{R}^2_{+}$ *be a rationalizable demand function which satisfies complementarity. Then* D *is continuous. Furthermore,* D *is rationalized by an upper semicontinuous, quasiconcave and weakly monotonic utility function.*

Nominal Income

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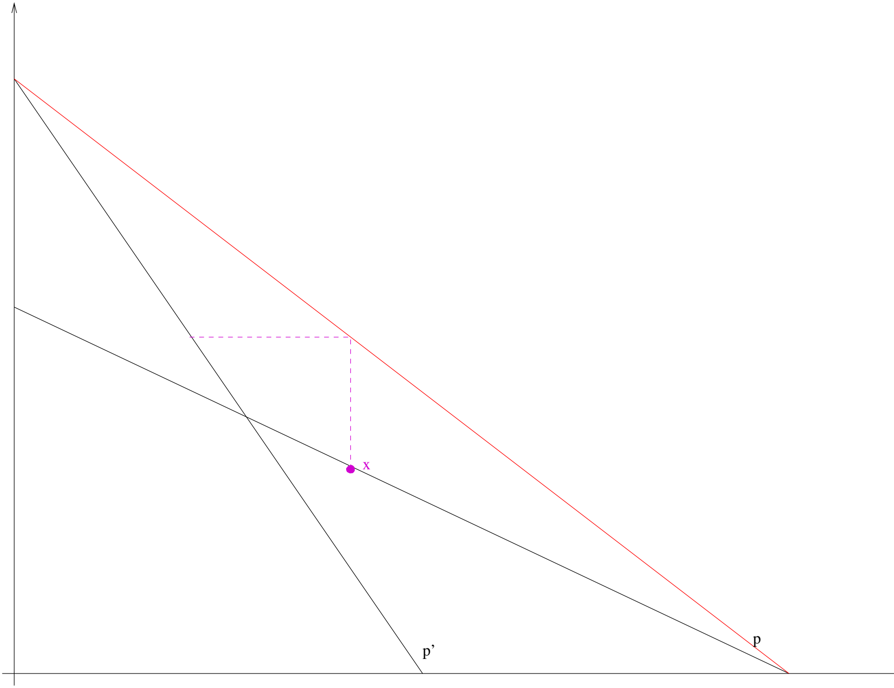
Let $D : \mathbb{R}^2_{++} \to \mathbb{R}^2_{+}$ be a rationalizable demand function which satisfies complementarity. Then D is continuous. Furthermore, D is rationalized by an upper semicontinuous, quasiconcave and weakly monotonic utility function.

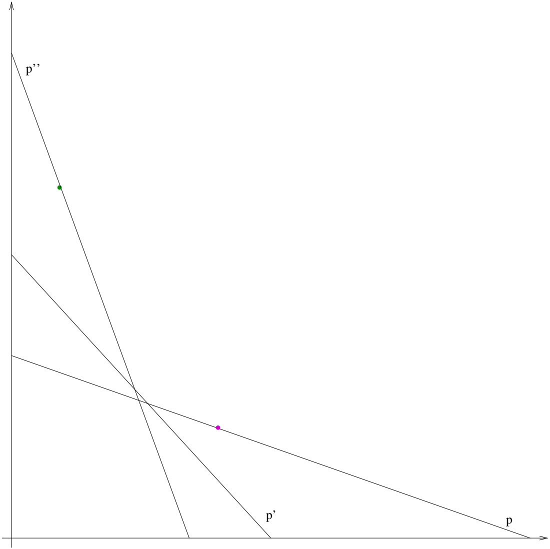
Remark

We extend the data to a demand, and then find it's rational. Difference from Afriat's approach of constructing a utility.

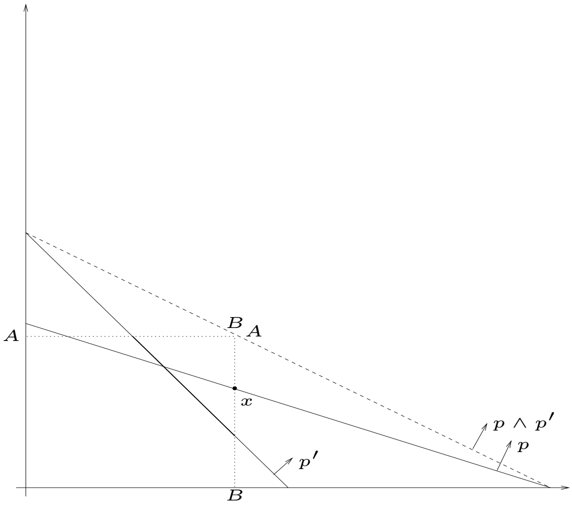
Theorem 1

Suppose we observe (p, x) and (p'', x'').
Find demand at prices p' (extend demand).


x
p'
p


p''
p'
p

Theorem (Observed Demand)



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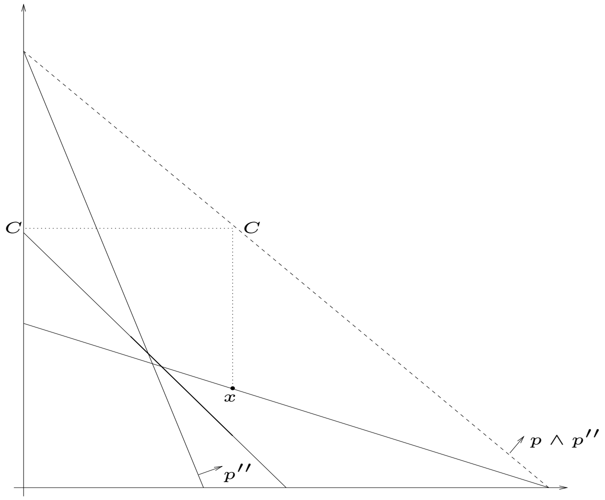
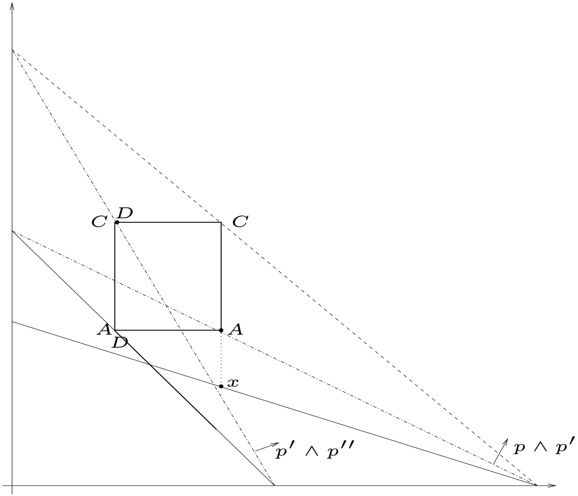


Theorem (Observed Demand)



Nominal Income

Let u be a C^2 utility.

$$m(x) = \frac{\partial u(x)/\partial x_1}{\partial u(x)/\partial x_2}$$

is the *marginal rate of substitution* of u at an interior point x.

Nominal Income

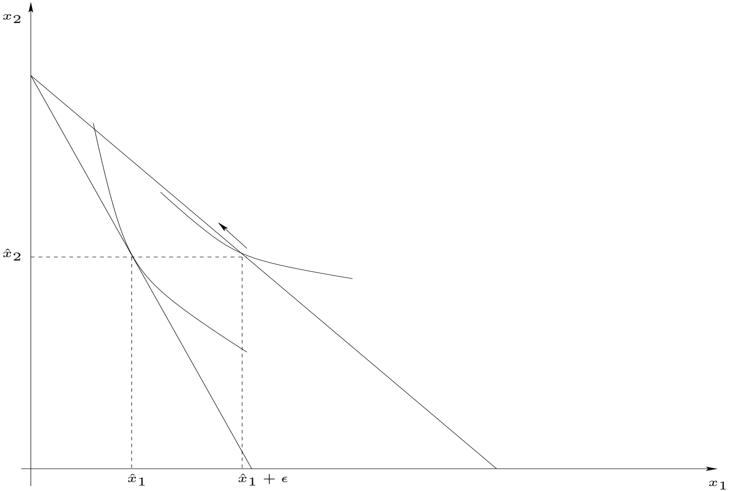
Let D a demand w/ interior range and monotone increasing, C^2, and strictly quasiconvex rationalization.

Theorem (Smooth Utility)

D satisfies complementarity iff

$$\frac{\partial m(x)/\partial x_1}{m(x)} \leq \frac{-1}{x_1} \quad \textit{and} \quad \frac{\partial m(x)/\partial x_2}{m(x)} \geq \frac{1}{x_2}$$

Theorem (Smooth Utility)



Nominal Income

Separability: $u(x, y) = f(x) + g(y)$.
Then complementarity iff f and g more concave than log.

Expect. util.: $\pi_1 U(x) + \pi_2 U(y)$.
Then complementarity iff RRA ≥ 1.

Analogous result for subst. due to Wald (1951) and Varian (1985).

Endowment Income

D satisfies *complementarity* if, for all (p, ω) and all p',

$$\left[D_1(p', p' \cdot \omega) - D_1(p, p \cdot \omega)\right] \left[D_2(p', p' \cdot \omega') - D_2(p, p \cdot \omega)\right] \geq 0. \tag{1}$$

D satisfies *weak complementarity* if, for every (p, ω), there is *one* price $p' \neq p$ satsfying (1).

Endowment Income

Let D be a rational demand.

Theorem (Endowment Model)

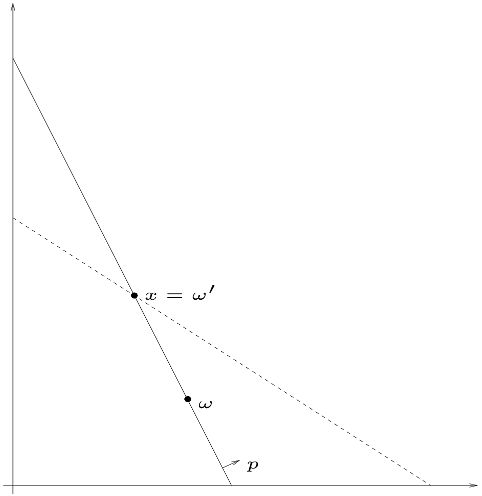
The following are equivalent:

1. *D satisfies complementarity.*
2. *D satisfies weak complementarity.*
3. *$\exists$ cont. strictly monotone, $f_i : \mathbb{R}_+ \to \mathbb{R} \cup \{\infty\}$, $i = 1, 2$, at least one of which is everywhere real valued ($f_i(\mathbb{R}_+) \subseteq \mathbb{R}$), s.t.*

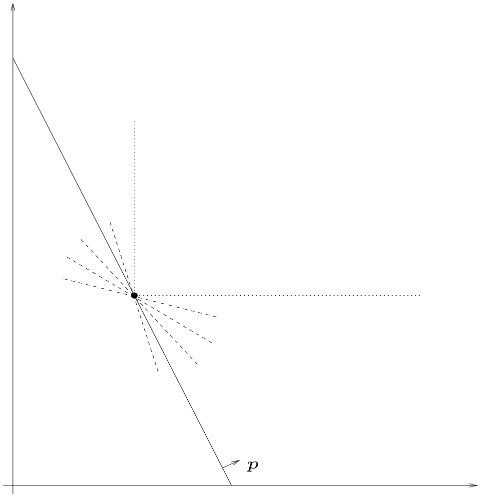
$$u(x) = \min\{f_1(x_1), f_2(x_2)\}$$

is a rationalization of D.

Endowment Income



Endowment Income



Conclusions: