27 Oct, 2020 **Ph 136a** E.S. Phinney

Applications of Classical Physics

Problem Set 4

Due **before 1:00pm** Wednesday 4 Nov, 2020

Readings: Readings for this week are to finish the readings on Chapter 4, if you have not done so, plus Sections 5.1-5.5 of Chapter 5 of Thorne & Blandford *Modern Classical Physics*, hereafter called TBMCP.

Non-Collaboration Problem Notice that problem 3 is a non-collaboration problem: you must solve this problem on your own (as if it were an untimed quiz), without discussing it with other students, or with the TA (see the collaboration and grading policy for how these will be graded and used: http://www.its.caltech.edu/~esp/ph136a/Policies.pdf)

Submitting your homework Please upload your completed homework solutions as a `pdf` file to Canvas. If that fails to work, you may instead email the file to the TA, `twang3@caltech.edu`, with the subject line `ph136 homework 4`. Note that Caltech email will reject attachment sizes larger than 10Mbyte, so be conscious of scanning parameters!

Homework Problems: (51 points total)

1. *Derivation of the occupation distribution function of the Canonical Ensemble* **(8 points: 4+4)**

   In section 4.4 of the text (around equations 4.18c-4.19b) it is argued that $\ln\rho = -\beta\mathcal{E} +$ constant, because $\rho = \Pi_a \rho_a$ and $\mathcal{E} = \sum_a \mathcal{E}_a$. Let us prove this mathematically.

   a) Suppose we have three functions f, g, and h, so that for all x and y,

   $$f(x)g(y) = h(x+y)\ . \tag{1}$$

   By taking partial derivatives with respect to x and then with respect to y, argue that for all x and y, we must have

   $$f'(x)/f(x) = g'(y)/g(y)\ . \tag{2}$$

   Show that this means we must have a constant β, such that

   $$f(x) = Ae^{-\beta x}, \qquad g(y) = Be^{-\beta y}\ . \tag{3}$$

   b) Use part (a) to prove that $\rho = Ce^{-\beta\mathcal{E}}$ and that $\rho_a = De^{-\beta\mathcal{E}_a}$.

2. **TBMCP Problem 4.2 (9 points: 3+3+3))** *Estimating Entropy*

   Pick any three of the 5 choices.

3. **TBMCP Problem 4.4 NONCOLLABORATION (11 points: 3+3+5)** *Entropy of a Thermalized Mode of a Field*

4. Do ONE of the following two problems (whichever is more interesting to you):

   a) **TBMCP Problem 4.13 (15 points: 9+6)** *Discontinuous Change of Specific Heat in Bose-Einstein Condensation in a Trap*

b) **TBMCP Problem 4.15 (15 points)** *Bose-Einstein Condensation in a Cubical Box*

5. **TBMCP Problem 5.2 (8 points: 3+3+2)** *Energy Representation for Nonrelativistic Classical Perfect Gas*

   In part (b), you need only check any two of the Maxwell relations, not all three.