2 March 2012 **Ay 21** E.S. Phinney

Galaxies and Cosmology

Problem Set 7

Due **in class**, 9 March 2012

Reading: Carroll and Ostlie (2nd edition), section 30.2 and Chapter 28.

Homework Problem:

1. **Microwave Background** The observed fluctuations in the microwave background on the last scattering surface on large angular scales are thought to be due in part to fluctuations in the gravitational potential of the dark matter.

   a) Consider a photon with energy $h\nu$ created by an atom at a point in a static gravitational potential ϕ ($\phi < 0$ in the potential wells where the mass density is high). Argue on the basis of energy conservation that the observed frequency will be Doppler-shifted by an amount $\delta\nu/\nu = \phi/c^2$ as it climbs out of the potential well. (Consider moving the atom out of the potential well and using it to absorb the photon, which you have stored.) Argue that the temperature measured from a distribution of photons will have fluctuations $\delta T/T = \phi/c^2$. Further details which you do not have to calculate, but will be needed in subsequent parts below: Note that your calculation was for a static potential. In an expanding universe, the potential is not static. When one more exactly in general relativity considers the redshift from a potential perturbation growing in an expanding universe, whose mean temperature is varying as $T \propto a^{-1} \propto t^{-2/3}$, one must chose a self-consistent choice of constant time slicing, and this contributes an initial temperature variation. Since clocks run slow by $(1 + \phi/c^2)$ in a gravitational potential ϕ, this choice of slicing adds a $\delta T_i/T = -\delta a/a = -2/3\delta t/t = -2/3\delta\phi/c^2$, so the correct final answer for the reduction in photon energy due to climbing out the the expanding potential is actually $\delta T/T = \delta\phi/3c^2$. Even this, however is not the *observed* temperature fluctuation, because potential wells occur where there are concentrations of dark matter. The deflection of photons increases the photon density, and hence temperature in these concentrations. A fully relativistic calculation evolving perturbations with a Harrison-Zel'dovich spectrum shows that $\delta \equiv \delta\rho_m/\rho_m = -2\phi$ and the observed $\delta T/T|_{obs} = -\delta/6$ (i.e. overdensities, which are intrinsically 'hot', appear as cold spots on the CMB sky, since the photons lose more energy climbing out of the potential).

   b) The measured fluctuations on angular scales larger than a few degrees are $\delta T \sim 30\mu$K. Estimate, using this fact and the result of the previous part, the amplitudes of the scale-invariant potential perturbations when they enter the horizon, $\delta\phi_H/c^2$ and the scale-invariant dark matter density perturbations $\delta\rho_m/\rho_m$ when they enter the horizon. (remember that we argued in class that the spectrum must be such that all perturbations enter the horizon with about the same potential perturbation).

   c) Estimate the comoving length scale r of a perturbation that would give rise to a $10^{12}M_\odot$ dark matter halo (like the Milky Way's).

   d) As described in class, dark matter perturbations which entered the horizon well before recombination grow after they enter the horizon: δ grows logarithmically with a during the radiation-dominated era, and linearly with a during the brief epoch between a_{eq} (when the matter and radiation mass-energy densities become equal) and recombination. Consequently, at the time of recombination, they have $\delta \simeq 51(\delta\phi_H/c^2)\ln(0.44c/(H(z_{rec})r)$ where $\delta\phi_H/c^2$ is the scale-invariant potential perturbation all fluctuations have when they enter the horizon. Evaluate this δ for the typical ("1σ") galaxy-scale perturbations of part 1c.

e) Consider the perturbations that may have formed the rare, first quasars and big galaxies. Suppose that these are "2σ" perturbations with dark mass $\sim 10^{12}M_\odot$ that reached maximum expansion when $z \sim 9$. Estimate (i) the redshift when they collapsed, and (ii) estimate the $\delta\rho/\rho$ they had at the epoch of recombination Is this consistent with the amplitudes you predicted for dark matter in part 1d?

f) Estimate the angular size in seconds of arc that those galaxy-scale perturbations would subtend on the surface of recombination/last scattering.

g) Estimate the age of the universe when CMB fluctuations which subtended angular scales $\sim 7^\circ$ at recombination, entered the horizon and came back into causal contact.

2. In class it was argued heuristically that gravity would not be able to grow density perturbations on scale r in baryons supported by pressure, if pressure waves could cross r in the gravitational collapse time, which is, up to a constant of order unity, also the expansion time in a critical ($k = 0$) universe $\sqrt{3/(8\pi G\rho)}$, i.e. the perturbation would not grow if $r/c_s \lesssim \sqrt{3/(8\pi G\rho)}$.

   a) Show that, up to constants of order unity, this is equivalent to the condition that the perturbed region's thermal energy exceeds its gravitational binding energy.

   b) Just after recombination, the gas temperature was $\sim$ 3000K. Estimate the minimum unstable scale for the baryons. Does this apply also to dark matter perturbations?

3. C& O problem 28.16

4. C& O problem 28.19