15 February 2012 **Ay 21** E.S. Phinney

Galaxies and Cosmology

Problem Set 6

Due **in class**, 28 February 2012

Reading: Carroll and Ostlie (2nd edition), sections 29.2 and 30.1.

Homework Problem:

1. Solve the Saha equation (C&O 29.101) for a universe made of pure hydrogen to find the temperature when half of the electrons and protons have combined to form neutral hydrogen.

2. C& O problem 29.23

3. C& O problem 29.17

4. a) Integrate the Planck formula for the number density n_γ and energy density u_γ of photons of energy $h\nu$ in frequency interval $d\nu$

$$N_\nu = 2\frac{4\pi\nu^2}{c^3}\frac{1}{\exp(h\nu/kT) - 1} \qquad (1)$$

   b) Repeat for the number density n_f and energy density u_f of any highly relativistic Fermion of spin $\hbar/2$ (this includes electrons, positrons, and all the three neutrino species ν_e, ν_μ and ν_τ and their antiparticles; though the neutrinos have only one spin state, so the prefactor $g = 1$ instead of $g = 2$ as for electrons and positrons),

$$N_\nu \simeq g\frac{4\pi\nu^2}{c^3}\frac{1}{\exp(h\nu/kT) + 1} \qquad (2)$$

5. In the early universe, the Universe was dense enough to be opaque to neutrinos. As the universe expanded, the density decreased until neutrinos no longer interacted with matter much and streamed freely (just as happened with photons after recombination, when $T < 3000$K). For $kT \gg 1$MeV, the reactions $\nu_e + \bar{\nu}_e \rightleftharpoons e^+e^-$ and $e^+ + e^- \rightleftharpoons \gamma + \gamma$ were in equilibrium, and as you saw in the previous problem, the number densities of photons, positrons and neutrinos were all similar. The cross-section for $\nu_e + \bar{\nu}_e \rightleftharpoons e^+e^-$ is energy dependent, roughly

$$\sigma_{\nu e} \sim 10^{-43}\text{cm}^2(E_\nu/1\text{MeV})^2, \qquad (3)$$

(compare this to the electron-photon Thomson scattering cross-section of $\sim 10^{-24}\text{cm}^2$).

Find the temperature T_ν at which the cosmic neutrino background decoupled from the rest of the matter. You should find that this occurs while the electrons and positrons are still numerous and relativistic. As the universe further cools, they annihilate into photons (but no longer into neutrinos), so the cosmic photon background ends up with a slightly higher temperature than the neutrino background: $T_\nu = (4/11)^{1/3}T_\gamma$, but you are not required to prove this, since it requires a little statistical physics you may not have seen yet.