17 Jan 2012 | **Ay 21** | E.S. Phinney

Galaxies and Cosmology

Problem Set 2

Due **in class**, 24 January 2012

Reading: Carroll and Ostlie (2nd edition), Section 1 of Chapter 26, Finish reading pages 1038-1069 of Chapter 27 if you haven't already done so.

Homework Problems:

1. **The Virial Theorem:**

   a) Repeat the derivation of the Virial Theorem in C& O section 2.4, but for an attractive "harmonic oscillator gravity," in which the force on m_i is

   $$\mathbf{F}_i = -\sum_{j\neq i} G m_i m_j (\mathbf{r}_i - \mathbf{r}_j) \ . \tag{1}$$

   Defining the potential energy to be that required to separate all the particles from a single point at the origin, show that the Virial Theorem becomes

   $$-2\langle K\rangle = -2\langle U\rangle \ , \quad \text{and hence } \langle E\rangle = 2\langle K\rangle = 2\langle U\rangle \ . \tag{2}$$

   b) Use this result to show that if energy is removed from such a system of harmonic oscillators, the kinetic energy decreases -i.e. it cools. This corresponds to matter with a positive specific heat, as have most familar materials around you.

   c) Consider two such systems in thermal contact with each other, one hot (high kinetic energies) and one cold (low kinetic energies). Thermodynamics guarantees that heat flows from the hot to the cold system. Show that the result is that the hot system cools down, while the cold one heats up, driving towards an equilibrium when the temperatures are equal.

   d) By contrast, use C& O's equations 2.46 and 2.47 to show that if energy is removed from a self-gravitating system, the kinetic energy *increases*, i.e. it heats up. This corresponds to a *negative specific heat*.

   e) Consider two self-gravitating stellar systems, one "hot" (high orbital velocities) and one "cold" (lower orbital velocities) which exchange energy with each other. Thermodynamics again applies, so heat flows from the hot to the cold system. Show that this means that the hot system gets hotter (more tightly bound) while the cold one gets colder (less tightly bound). No equilibrium is reached!

2. **Discover Dark Matter Yourself:**

   Here you work through a modern version of the classic papers (1933, 1937) in which Caltech's Fritz Zwicky discovered dark matter. For this problem, you will need to be able to read an ascii data file into a computer, plot it, and do arithmetical operations/statistical calculations with the numbers.

   Figure 1 shows the central $0.5° \times 0.9°$ of the cluster of galaxies in the constellation Coma Berenices, dominated by the two giant ellipical galaxies NGC 4889 ($cz = 6505$km s^{-1}) and

NGC 4874 ($cz = 7108\text{km s}^{-1}$), separated by $0.12°$. $0.66°$ away from NGC 4874, off the lower right of the image on the next page, lies the galaxy NGC 4839 ($cz = 7442\text{km s}^{-1}$). Figure 2 shows an image of the $10' \times 10'$ field centered on NGC 4839 (the largest, brightest galaxy, at the center).

The table http://www.its.caltech.edu/~esp/ay21/ps1coma_center.txt gives the measured radial velocities of 270 galaxies in the $48' \times 25'$ field (roughly that of Figure 1) centered between the two giant elliptical galaxies (NGC 4889 is galaxy 65 in the table, and NGC 4874 is galaxy 124) that define the center of the cluster. The table http://www.its.caltech.edu/~esp/ay21/ps1coma_outrv.txt gives the measured radial velocities of 34 galaxies in the $10' \times 10'$ field centered on NGC 4839 shown in Figure 2 (galaxy 22 in the list is NGC 4839). In the tables, the first column is galaxy number. The second column is each galaxy's total b magnitude (astronomers' way of specifying the flux in blue light above a certain surface brightness; smaller numbers are brighter, so you can see that the two giant ellipticals and NGC 4839 are by far the brightest in their respective tables), and the third column is the speed of light c times the redshift z, cz, i.e. the 'radial velocity'. The authors of the table estimate the radial velocity errors to be about 65km s^{-1}.

Notice that most of the galaxies have about the same redshift, but some have very discrepant redshifts. These are galaxies far behind or well in front of the cluster. There will also be some galaxies just in front or just behind, which are beginning to fall into the cluster.

a) Give your best estimate of the distance to the Coma cluster (adopt $H_0 = 70\text{km s}^{-1}\text{Mpc}^{-1}$). Discuss and justify the method by which you eliminated foreground and background galaxies and included only cluster members.

b) Give your best estimate of the dispersion (standard deviation, $\sigma = \sqrt{(\sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1))}$) in radial velocity of galaxies *that are members of the Coma cluster*. Discuss and justify the method by which you eliminated foreground and background galaxies and included only cluster members. Do you get the same dispersion in the central region and the outlying NGC 4839 region?

c) More than half the bright galaxies that belong to the Coma cluster lie within 1.3 degrees of the center. Convert this to radius perpendicular to the line of sight, in Mpc.

d) If galaxies in the Coma cluster were mistakenly assumed to be expanding with the rest of the universe (i.e. their *individual* velocities were converted to distance using the Hubble constant), what typical radial distance error would we be making? [1]

e) Taking your dispersion from part (b) and your estimate of the cluster radius from part (c), use the Virial theorem to estimate the total gravitational mass of the Coma cluster, in solar masses ($1M_\odot = 1.99 \times 10^{30}\text{kg}$).

f) In the infrared K-band, the total K-band flux of all the galaxies in the Coma cluster gives the total K-band luminosity as $2.3 \times 10^9 L_{\odot K}(D/\text{Mpc})^2$, where D is the distance of the Coma cluster, which you determined in part (a) An old population of stars like that in most Coma cluster galaxies, with the standard 'Kennicutt' distribution of stellar masses, has a mass of 0.73 $M_\odot$ (solar masses) per $L_{\odot,K}$ (solar luminosity in K-band) it emits (i.e. the K-band mass-to-light ratio is $0.73M_\odot/L_{\odot,K}$). Estimate the total mass in stars in the galaxies of the Coma cluster. What percentage is this of the total mass you estimated in part (e)?

[1] The fact that this is much larger than the answer to the previous question is known as the "finger of God effect".

g) Pages 1066-1068 of Caroll & Ostlie estimate the mass of hot gas in the Coma cluster from the gas's X-ray emission. What fraction is this of the total cluster mass you estimated in part (e)?

h) Calculate the kurtosis (defined as $\sum_i (x_i - \bar{x})^4/\sigma^4 - 3$, where $\bar{x}$ is the mean of the x_i, and σ the standard deviation as in part (b)) of the radial velocities of Coma cluster members (i.e. excluding the outliers) in the central and in the NGC 4839 region data tables. The kurtosis of a Gaussian (Maxwellian) distribution is 0, and that of a top-hat distribution (uniform distribution in some range, dropping suddenly to 0 probability outside that range) is -1.2. You should find that the velocity distribution is close to Gaussian in the central field, but very non-Gaussian in the NGC 4839 field.

i) Can you suggest a reason for the result of part (h)? Hint 1: Gaussian distributions are the result of the central limit theorem, i.e. result when masses like galaxies have had many momentum-exchanging encounters with each other. Compare the time it takes a galaxy to cross the Coma cluster with the age of the Universe (13.7 Gyr). Hint 2: Big clusters like Coma grow as smaller clusters fall into them and are disrupted by their tidal gravitational field.

3. **Dynamical Friction:** The oldest globular clusters are composed of stars 1.25×10^{10} years old. Approximating the Milky Way as spherical, with a flat rotation curve, estimate the greatest distance from which globular clusters could have spiraled into the nucleus of the Milky Way due to dynamical friction. Use $2 \times 10^6 M_\odot$ for the cluster's mass. Compare your answer to the size of the Milky Way's central bulge.

Figure 1: The central 0.5 by 0.9 degree of the Coma cluster of galaxies. The two brightest galaxies are (left) NGC 4889 and NGC 4874, separated by 0.12 degrees. Image from Robert Lupton and the Sloan Digital Sky Survey.

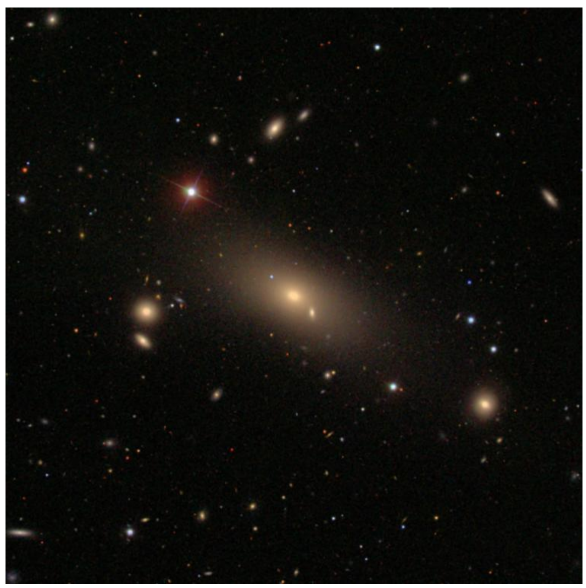

Figure 2: The 10 by 10 arcmin region of the Coma cluster centered on the galaxy NGC 4839, lying 0.66° southwest of the center of the cluster. Image from SDSS DR8.