10 Jan 2012 **Ay 21** E.S. Phinney

Galaxies and Cosmology

Problem Set 1

Due **in class**, 17 January 2012

Reading: Carroll and Ostlie (2nd edition), Section 4 of Chapter 2, Sections 1, 2 and 4 of Chapter 25, and pages 1038-1069 of Chapter 27,

Homework Problems:

1. **Finding and interpreting data on a galaxy:** Use the NASA Extragalactic Database (NED) `http://ned.ipac.caltech.edu` to look up the galaxy NGC 2639.

   a) Click on the Images box in NED's Detailed Data on NGC 2639. Browse through the images until you find one which you feel will be useful for determining the Hubble type (see Fig 25.1 on p 943 of Carroll & Ostlie), and do your best to determine the Hubble type. Hint: if you download a FITS file, the extreme dynamic range of astronomical images may require manipulation of the map between photons/pixel and brightness on your computer screen to show the features of the galaxy best. A free tool to do this is DS9: `http://hea-www.harvard.edu/RD/ds9/`. Try the "scale" menu and the "zscale" therein.

   b) Estimate the inclination angle between the vector normal to the disk and the vector from earth to NGC 2639, in degrees (90 for edge-on disk, 0 for face-on)

   c) The observed radial velocity of the stars in the disk as a function of position along the center of the long axis of NGC 2639's disk are shown in figure 1. If you consider stars orbiting in a circle, the observed radial velocity in the orbit along the major axis of the circle's projected ellipse is less than the orbital velocity unless the inclination is 90 degrees (it is 0 if the inclination is 0 degrees!). Figure out the inclination correction for the inclination you estimated in the previous part, and apply it to the data, to determine

      i. the heliocentric radial recession velocity of the center of NGC 2639, and

      ii. the maximum rotational orbital velocity about that center of the stars in NGC 2639.

      iii. The distance to NGC 2639. Pretend both it and the sun are cosmic observers (actually the sun is not, moving at a few hundred km/s around the Milky Way, which in turn is falling towards the Virgo cluster. This introduces only a 10% distance error, which you may neglect for the purposes of this problem. NGC 2639 is presumably moving at a similar, but unknown, velocity relative to cosmic observers). Use a Hubble constant $H_0 = 73\text{km s}^{-1}\text{Mpc}^{-1}$.

   d) Go to the Sloan Digital Sky Survey server, http://skyserver.sdss3.org/dr8/en/tools/chart/navi.asp and enter the coordinates (in decimal equatorial) that NED gives you for NGC 2639. You should get an image, with sky-subtracted magnitudes (integrating flux from each solid angle over solid angle out to the limits where the galaxy is detectable) in 5 color bands (u,g,r,i,z). [1] The absolute magnitude of the sun in those bands is respectively (6.55, 5.12, 4.68, 4.57 and 4.60). Calculate the luminosity of the galaxy in $L_{\odot,g}$ —i.e. how many suns would be required to produce the NGC 2639's luminosity in g band?

[1] If you are curious about what exactly the filter bands As you might have guessed, g band covers green and blue light, and r band covers red and yellow light.

e) The Tully-Fisher diagram of Carroll & Ostlie Figure 25.10 (page 954) was made in the days when it was believed that $H_0 = 50\text{km s}^{-1}\text{Mpc}^{-1}$. How should the vertical axis be changed to reflect the modern value of $H_0 = 73\text{km s}^{-1}\text{Mpc}^{-1}$?

f) The B-band apparent magnitude of NGC 2639 is is $m_B = 12.2$ (same as m_g). Plot NGC 2639 on the Tully-Fisher relation shown in C& O Figure 25.10, corrected to the modern value of H_0. NGC 2639 is one of the galaxies plotted. Can you figure out which point it is?

g) All the detectable light of NGC 2639 is contained within 60 seconds of arc of the center[2] (measured along the major axis). Assuming the rotation curve of figure 1 continues flat to this distance, estimate the mass in NGC 2639 within this radius.

h) Calculate the g-band mass-to-light ratio of NGC 2639 within the 60" radius, in solar units ($M_\odot/L_{\odot,g}$).

2. **Measure a redshift yourself:** Figure 2 shows the spectrum of a (real) galaxy, randomly selected from the millions in the SDSS database. You are to identify the spectral lines and figure out its redshift. The rest (laboratory-measured) wavelengths of some famous strong spectral lines commonly observed in absorption in the atmospheres of the old sun-like stars which make up galaxies like this are given in the table below (not all of these may be easily apparent in the spectrum). See if you can identify *at least* five absorption lines in the spectrum, and determine the redshift of the galaxy. [hint: one approach is to measure the wavelengths of the absorption features in the spectrum, make a matrix of the ratios of all those wavelengths with those in the table below, and look for a commonly repeated number, which will be $1 + z$. Or you may think of faster methods.]

ion	wavelength (Å)
Ni I	3524
Fe I	3815, 3820,3825
Mg I	3832, 3838
Ca II, K	3934
Ca II, H	3968
Fe I, G	4308
Ca I	4227
Hγ	4340
Hβ	4861
Mg I	5167, 5172, 5183
Na I, D	5890, 5896
Hα	6563

[2] remember that there are 60×60 arcsec in one degree.

3. **Build-a-galaxy:** Having been deified, you decide one day to create a galaxy. As gods have high standards, you want your galaxy to be eternally unchanging. You realize that the positions of the stars or dark matter particles making up the galaxy will be changing all the time, so this cannot be strictly true, but you want the mean density field and mean gravitational potential, averaged over volume elements containing hundreds of particles, to be time-independent. Having been deified before completing ACM 95 and Ph 129, you are not quite omniscient, so decide as a first attempt, to create an infinite plane-parallel galaxy, whose mathematics is much simpler than a fully 3-D galaxy.

   a) As you put particles down, you must choose both where to put them, and what velocities to give them. This means, that averaged over a volume element, each point in space has a mass density, and a velocity distribution function. For simplicity assume that all the particles you use to build your galaxy have the same mass m. Define the number of particles per unit volume in range of height dz and range of velocity dv_z to be $f(z, v_z)dz\, dv_z$. Show that the mass density is

   $$\rho(z) = \int_{-\infty}^{\infty} m f(z, v_z) dv_z \; . \tag{1}$$

   b) Realize that your desire to make an eternally unchanging galaxy makes this a tricky problem: if you put all the particles near a thin plane, but give some of them large v_z, they will fly outwards, thickening the plane with time. Conversely, if you make a thick plane, but give some of the particles too small velocities, they will fall back to make a denser thin plane within your thick one. So you have to get $f(z, v_z)$ just right. You remember, however, that if the gravitational potential ϕ is not changing with time, as you wish, that the *energy* $E = \phi + v_z^2/2$ of each particle will be independent of time. Argue that if you choose $f(z, v_z) = f(E)$, the density and velocity distributions will automatically be such as to make your galaxy eternally unchanging. $f(E)$ is an arbitary function. Different $f(E)$ correspond to different density distributions (see equation 1).

   c) However, to determine the density distribution $\rho(z)$ resulting from some choice $f(E)$ (or vice-versa) is still a hard problem. Show that you must solve the following nonlinear integro-differential equation for ϕ

   $$\frac{d^2\phi}{dz^2} = 4\pi G m \int_{-\infty}^{\infty} f(\phi + v_z^2/2)\, dv_z \; . \tag{2}$$

   d) Before tackling this, make a rough estimate —suppose you want most of the stars of your disk to be spread over a scale height H above and below the midplane, and the disk to have a total surface density (mass/area) Σ.

      i. Show that the density at the midplane must then have $\rho_0 \approx \Sigma/(2H)$.

      ii. Show that the potential near the midplane of the disk ($z \ll H$) is $\phi \approx 4\pi G\rho_0 z^2$, and far from the midplane ($z \gg H$) is is $\phi \approx 2\pi G\Sigma z$ (hint: use Gauss's theorem and pill box integration).

      iii. By considering the heights of turnaround of particles in such potentials, show that the typical particle velocity near the midplane must be

      $$v_z \approx \sqrt{8\pi G\rho_0} H \tag{3}$$

      This gives you the rough magnitude of velocity, but not the precise distribution of velocities, which will of course depend on the precise vertical density distribution you choose for your eternal galaxy.

e) Thinking back to thermal physics and Boltzmann factors, you decide to see if you can solve equation 2 for

$$f(E) = \frac{\rho_o}{m\sqrt{2\pi\sigma^2}} e^{-E/\sigma^2} \; . \tag{4}$$

Show that in this case equation 2 becomes

$$\frac{d^2(\phi/\sigma^2)}{dz^2} = \frac{4\pi G\rho_0}{\sigma^2} e^{-\phi/\sigma^2} \; , \tag{5}$$

and that at every point, the vertical velocity distribution is a Maxwellian (aka Gaussian) with height-independent dispersion σ.

f) Show that the boundary conditions are $\phi(0) = 0$ and $d\phi/dz(0) = 0$.

g) Define new dimensionless variables $\psi = \phi/\sigma^2$ and $\zeta = z/H$, where $H = \sigma/\sqrt{8\pi G\rho_0}$ (compare to your estimate equation 3). Show in these variables, equation 5 becomes

$$2\frac{d^2\psi}{d\zeta^2} = e^{-\psi} \; , \qquad \psi(0) = 0 \; , \qquad d\psi/d\zeta(0) = 0 \; . \tag{6}$$

h) Autonomous (ζ does not appear explicitly) nonlinear differential equations such as this can be reduced in order by substituting $u(\psi) = d\psi/d\zeta$. Use the chain rule to show that with this substitution, equation 6 and its boundary conditions become the first order equation

$$\frac{du^2}{d\psi} = e^{-\psi} \; , \qquad u(\psi = 0) = 0 \; . \tag{7}$$

i) This first order equation can be solved by direct integration. Show that the solution is $u^2 = 1 - \exp(-\psi)$.

j) Now go back to the original dimensionless variables. Show that $u = d\psi/d\zeta = \sqrt{1 - \exp(-\psi)}$, and show that this can be solved by direct integration, with the result

$$\zeta = 2\tanh^{-1}\sqrt{1 - \exp(-\psi)} \; . \tag{8}$$

k) Finally show that this gives

$$\rho(\zeta)/\rho_0 = e^{-\psi} = 1 - \tanh^2(\zeta/2) = \mathrm{sech}^2(\zeta/2) \; , \tag{9}$$

where $\zeta = z/H$, $H = \sigma/\sqrt{8\pi G\rho_0}$. Sketch $\rho(z)$ versus z for your new eternally unchanging infinite plane galaxy. This famous self-consistent solution was first found by Lyman Spitzer Jr. in 1942. Of course, as a deity, you might prefer some other $f(E)$, but you may find computing the resulting $\rho(z)$ to be harder.

Not required -for fun As a deity, you are omnipotent, but must still obey the laws of physics. Can you make the vertical density distribution have any form you want, and still be time-independent with a suitable choice of velocity distribution? Can you either prove you can, or find a density distribution that couldn't be made time-independent?

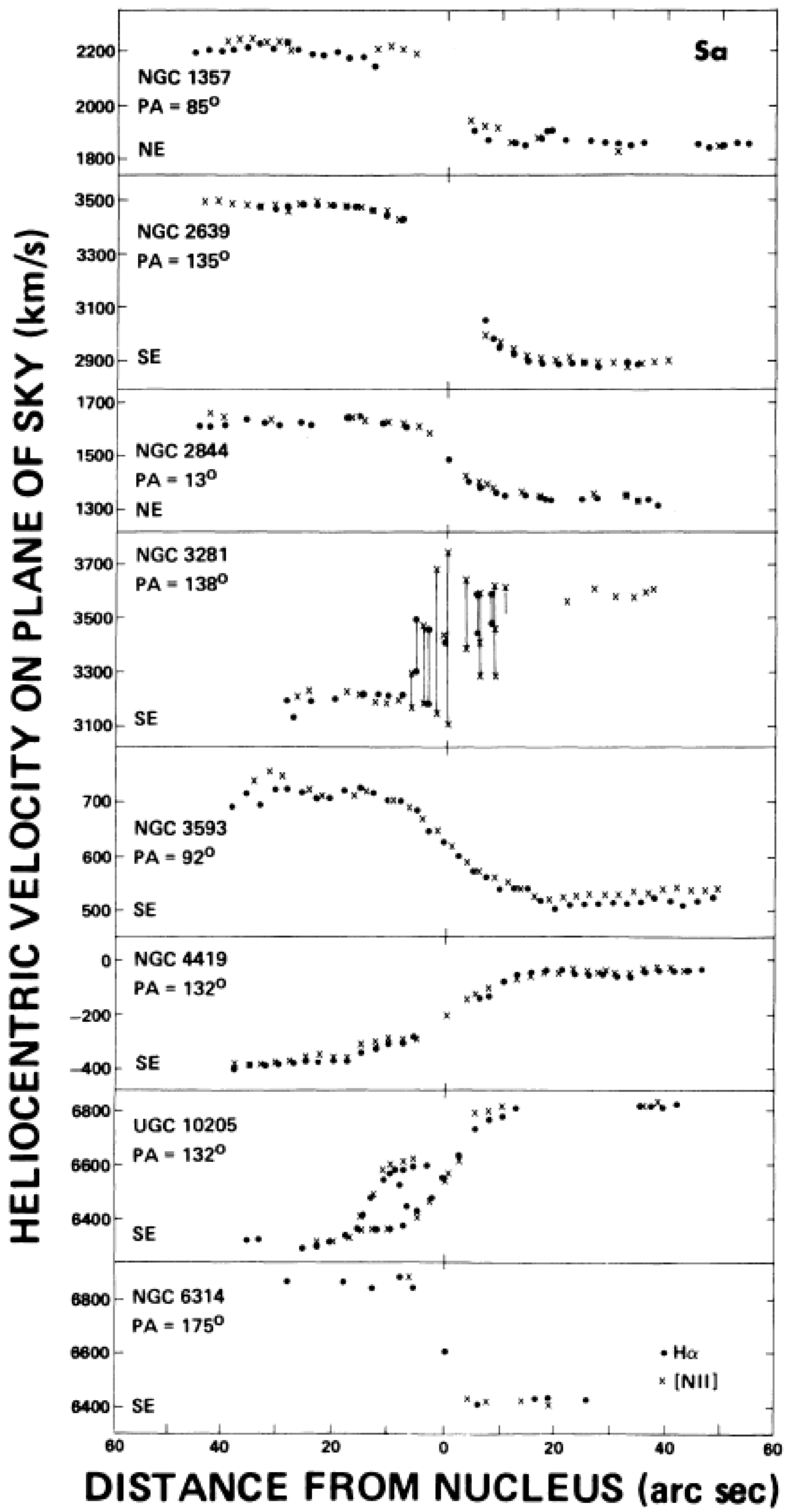

Figure 1: The radial velocity (relative to the sun) versus sky position (in arcseconds from the galaxy center) along the observed major axis for several galaxies, including NGC 2639.

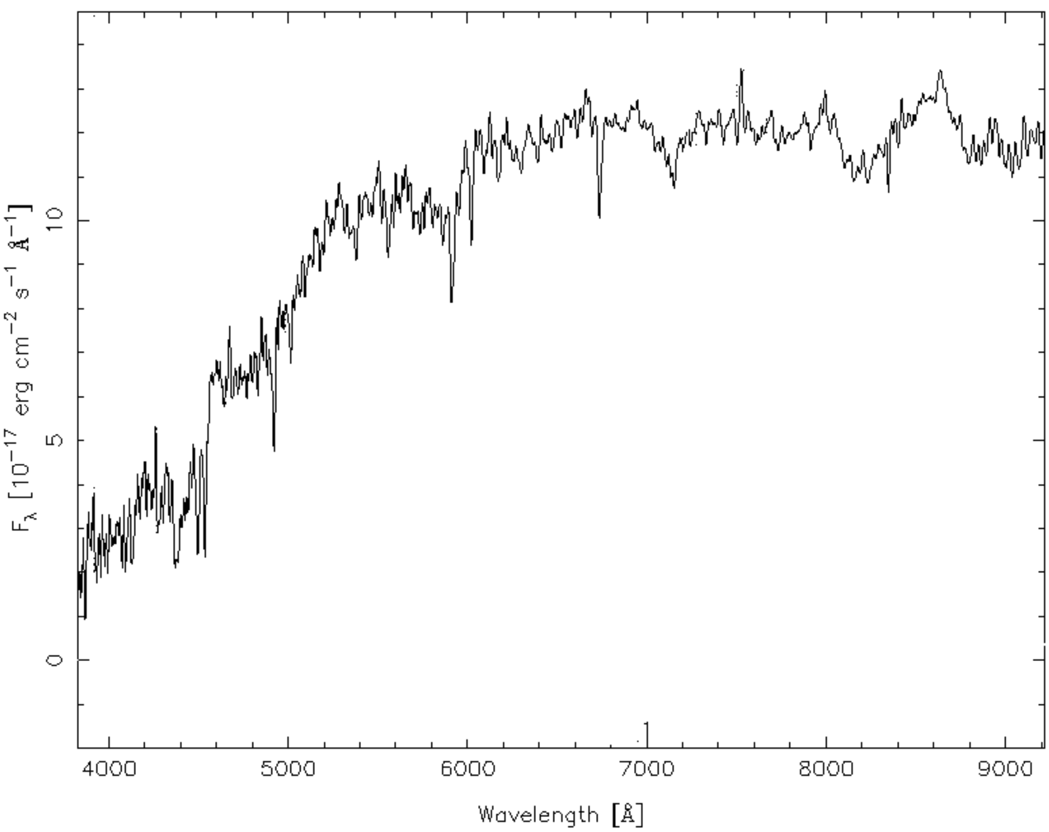

Figure 2: The spectrum of an elliptical galaxy. The vertical axis shows the flux of radiation per unit of wavelength. The horizontal axis shows the wavelength in Ångstroms (1Å=0.1nm), blue on the left, red and then near infrared on the right.