Ay 20 Basic Astronomy and the Galaxy
Problem Set 4

Tim Morton, Varun Bhalerao & Sterl Phinney

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1 The 51 Pegasi system

a) Mass Refering to Figure 7.7 in C&O, we can see that the points on the log–log plot of L versus M is a straight line - thus mass and luminoisty must be related by a power law. We can fit a straight line to it by reading off any two points. We see that (log M, log L) = (-0.75, -3) and (1.25, 5) are points on the line. Also, the the line passes through (0,0). This tells us that if we measure L in units of solar luminosity ($L_\odot$) and mass in units of solar masses $M_\odot$ then we have:

$$\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^4 \tag{1}$$

Substituting $L=1.3\,L_\odot$ gives $\boxed{M=1.1\,M_\odot}$.

b) Radius To calculate the radius, we use the luminosity and effective temperature. We have,

$$L = 4\pi R_*^2 \sigma T_*^4 \tag{2}$$

Substituting $L_\odot=3.8\times10^{33}\,\mathrm{erg\,s^{-1}}$, $T_*=5660\,\mathrm{K}$, we get:

$$R_* = 8.3\times10^{10}\,\mathrm{cm} = 1.2\,R_\odot \tag{3}$$

c) Variation of Hβ wavelength The wavelength of the Hβ line measured in air is λ_0 4861.34 Å. We see from Figure 1 in the problem set that v_r varies from -60 m/s to +55 m/s. We can then calculate the variation in range of variation of λ by using the formula,

$$\Delta\lambda = \frac{v}{c}\lambda \tag{4}$$

Hence λ varies from $\lambda_0 - 9.7\times10^{-4}$ Å to $\lambda_0 + 8.9\times10^{-4}$ Å, corresponding to a total variation of $\boxed{0.19\,\mathrm{pm}}$.

d) Selecting a good line The full width at half maximum (FWHM) of the Hβ line in the solar spectrum is about 0.1 Å. To measure velocities of ∼5 m/s, we have to measure $\Delta\lambda \sim 10^{-4}$ Å. Hence we have to centroid the line to 10^{-4} times its width, which will be prone to a lot of errors. A better choice will be to use one of the narrower lines in the spectrum, which are much sharper. Note that the Hβ line is broad and hence is the main contributor to the signal - resolving narrow lines requires high resolution spectroscopy and a high signal–to–noise ratio.

e) The planet

i. Semi major axis We use Kepler's law to calculate the semimajor axis of the orbit:

$$\frac{P^2}{a^3} = \frac{1}{M_* + M_p} \tag{5}$$

where P is the period in years, a is the semimajor axis in AU, M_* is mass of the star, M_p is the mass of the planet, both measured in solar masses. Neglecting the mass of the planet, we get:

$$a = \sqrt[3]{1.1 \times \left(\frac{4.231}{365.25}\right)^2} = 0.053\,\mathrm{AU} = 7.9 \times 10^{11}\,\mathrm{cm} \tag{6}$$

ii. Mass of the planet To estimate the mass of the planet, we use the modulations in the star's velocity. We know that:

$$M_* v_* = M_p v_p \tag{7}$$

$$v_p = \frac{2\pi a}{P} = 1.4 \times 10^7\,\mathrm{cm\,s^{-1}} \tag{8}$$

Substituting this value in Equation 7 we get,

$$M_p = 1.1\,M_\odot \times \frac{(55 - (-60)) \times 100}{2}\,\mathrm{cm\,s^{-1}} \times \frac{1}{1.4 \times 10^7\,\mathrm{cm\,s^{-1}}} = 4.7 \times 10^{-4}\,M_\odot = 9.3 \times 10^{29}\,\mathrm{gm} \tag{9}$$

f) Temperature of the planet We assume that the planet rotates rapidly and the surface is at a uniform temperature. The flux from the star at the planet's orbit is given by,

$$F = \frac{L_*}{4\pi a^2} \tag{10}$$

The energy absorbed by the planet is then:

$$E_{in} = 0.5 \times \pi R_p^2 F = \frac{\pi R_p^2 L_*}{8\pi a^2} \tag{11}$$

This energy is re–radiated as a blackbody:

$$E_{out} = 4\pi R_p^2 \sigma T_p^4 \tag{12}$$

Solving Equations 11 and 12 together we get:

$$T_p = \sqrt[4]{\frac{L_*}{4\pi a^2} \times \frac{\pi R_p^2}{2} \times \frac{1}{4\pi R_p^2 \sigma}} = 1100\,\mathrm{K} \tag{13}$$

In reality, the planet is orbiting so close to the star, that it is "tidally locked" to the star, just as moon is locked to the earth. So, the same surface of the planet faces the star all the time. At a place on the planet which has high noon, the starlight is incident normally on the "ground". The temperature can then be calculated by equating the energy absorbed and re–radiated by a unit area:

$$0.5 \times \frac{L_*}{4\pi a^2} = \sigma T^4 \tag{14}$$

This gives a temperature $\sqrt{2}$ times higher than the previous case: $T_p \sim 1550\,\mathrm{K}$

2 Finding Exoplanets

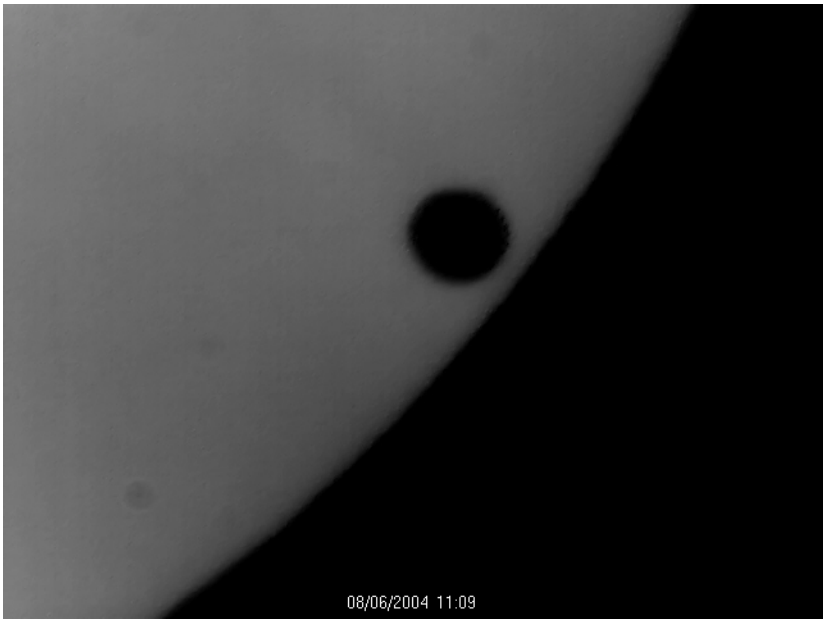


Figure 1: Image of venus transiting the sun, 8 June 2004. (Webcam on 6 inch newtonian telescope with solar filter)

a) Detecting Jupiter Since we are searching for a planet around a distant star, we can consider parallel rays coming from the star. If we could resolve the disc of the star, we would see the transit simply as a dark dot on the stars disc. (For example see Figure 1 for an image of venus transiting the disc of the sun.) For a distant star, we cacnt resolve the disc, so we will just note a drop in the flux of the star. The fractional decrease is simply given by the ratio of the areas:

$$f = \frac{\pi R_*^2}{\pi R_J^2} = 0.01 \tag{15}$$

Where we have used $R_J = 7.14 \times 10^9\,\mathrm{cm}$. A useful website to get such constants is http://www.astro.wisc.edu/~dolan/constants.html. Here, we have also neglected any emission from Jupiter itself. If jupiter was also radiating significantly, there would be another correction to this, given by:

$$\Delta f = \frac{T_J^4}{T_\odot^4} \tag{16}$$

Let us again assume an albedo of 0.5. We can either estimate T_J or we can directly use Equation 13, substituting $L_* = 4\pi R_\odot^2 \sigma T_\odot^4$, to get:

$$\frac{T_J^4}{T_\odot^4} = \frac{R_\odot^2}{8a^2} \approx 10^{-7} \tag{17}$$

Hence, we can ignore this correction in calculating the fractional decrease in brightness.

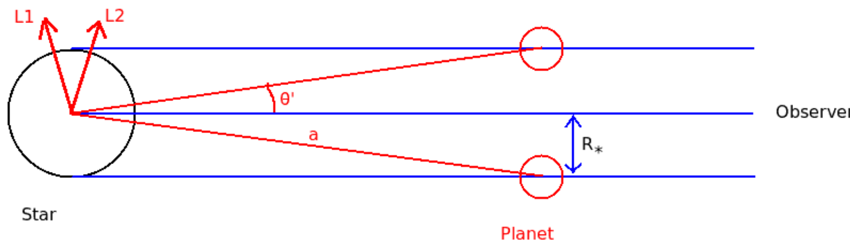


Figure 2: Range of angles in which an observer will see a transit/occultation

b) Probability of finding Jupiter Refer to Figure 2. The planet will occult the star only if the angular momentum vector is in the range L1 to L2, which means $\theta' < sin^{-1}\left(\frac{R_\odot}{a}\right)$ where $R_\odot$ is the radius of the star (in this case the sun), and a is the star–planet separation. Thus we can calculate the probability of the occultation:

$$P = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=\frac{\pi}{2}-\theta'}^{\frac{\pi}{2}-\theta'} sin(\theta)d\theta \tag{18}$$

$$P = -\frac{1}{2}\left[cos\left(\frac{\pi}{2}+\theta'\right) - cos\left(\frac{\pi}{2}+\theta'\right)\right] \tag{19}$$

$$P = -\frac{1}{2}\left[-sin(\theta') - sin(\theta')\right] = sin(\theta') = \frac{R_\odot}{a} \tag{20}$$

Substituting $a = 1\,\text{AU}$ we get $\text{P} = 8.9 \times 10^{-4}$.

By a similar argument, the probability of a grazing transit will be $2R_p/a$, or $\sim 1.8 \times 10^{-4}$.

c) Detecting 51 Pegasi First let us calculate the radius of 51 Peg:

$$R_* = \sqrt{\frac{L}{4\pi\sigma T^4}} = 8.3 \times 10^{10} cm = 1.2\, R_\odot \tag{21}$$

Then we assume that the orbiting planet has the same radius as jupiter, and proceed like part (a) to get,

$$\Delta f = 7.4 \times 10^{-3} = 0.74\% \tag{22}$$

Again, we have to verify that the correction due to the planets emission is small. Indeed we see that $T_J^4/T_*^4 = 1.4 \times 10^{-3}$, which is negligible.

d) Probability of finding 51 Pegasi–like systems Again proceeding like part (b), we use Equation 20 to get,

$$P_{51\,Peg} = \frac{R_*}{a_*} = 0.1 \tag{23}$$

3 Hydrogen gas

This is a straightforward application of the Boltzmann Equation. For having only 1% atoms in the first excited state,

$$10^{-2} = \frac{2 \times 2^2}{2 \times 1^2} e^{-\left[\frac{-13.6\,\mathrm{eV}}{2^2} - \frac{-13.6\,\mathrm{eV}}{1^2}\right]/kT} \tag{24}$$

Hence we get $T = 2.0 \times 10^4$ K. For having 10% atoms in the excited state, we get $T = 3.2 \times 10^4$ K.

4 Solving the Saha Equation

See separate pdf of hand-written solution below:

5 Giant stars and spectral type

The key to this problem is understanding the meaning of spectral type as a set of particular line ratios. Then, using the Saha equation, the argument is straightforward: two parameters are free (free electron density and temperature), and thus a giant star with a low density will be cooler to have the same line ratios.

PROBLEM SET 6

P1 : CARROLL & OSTLIE 8.9

$$n_e V = N_{II}$$

$$\rho = 10^{-6}\ \text{kg m}^{-3} = 10^{-9}\ \text{g cm}^{-3}$$

$$N_t = \frac{\rho V}{m_p}$$

a)
$$\frac{N_{II}}{N_t - N_{II}} = \frac{2 \times 1}{n_e \times 2}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT}$$

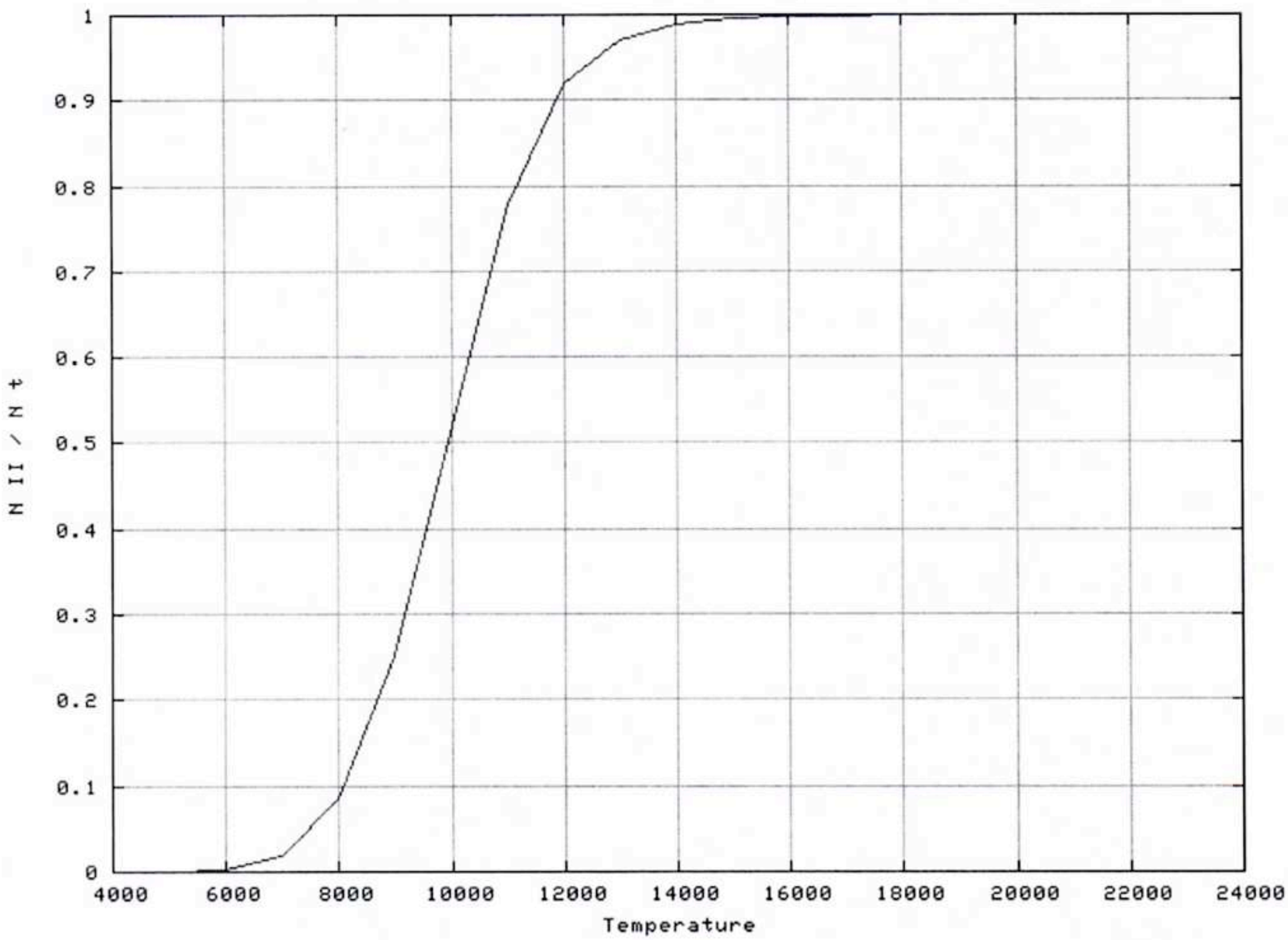
but
$$n_e = \frac{N_{II}}{V} = \frac{N_{II}\,\rho}{m_p N_t}$$

$\therefore$ we get

$$1 = \left(\frac{N_t}{N_{II}} - 1\right)\frac{N_t}{N_{II}}\left(\frac{m_p}{\rho}\right)\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT}$$

$$\therefore \left(\frac{N_{II}}{N_t}\right)^2 = \left(\frac{m_p}{\rho}\right)\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_I/kT}$$

$$- \frac{N_{II}}{N_t}\left(\frac{m_p}{\rho}\right)\left(\frac{2\pi m_e kT}{h^2}\right) e^{-\chi_I/kT}$$



1) b) we have a quadratic in $x = \frac{N_{II}}{N_t}$

expressing as $ax^2 + bx + c = 0$, we get

$$a = 1$$

$$b = \frac{m_p}{\rho}\left(\frac{2\pi m_e k T}{h^2}\right)^{3/2} e^{-\chi_I/kT}$$

$$c = -b$$

since $x = \frac{N_{II}}{N_t}$ is > 0,

$$x = \frac{-b + \sqrt{b^2 + 4b}}{2}$$

: see graph above. It matches well with 8.8