

Ay 20 Basic Astronomy and the Galaxy

Problem Set 3

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1 Barnard's star

a) Radial velocity We observe the H_α in air at a wavelength of $\lambda_0 = 656.281$ nm. The radiation from the star is seen at a wavelength of $\lambda = 656.034$ nm in air. The radial velocity of the star is,

$$v_r = c \times \frac{\lambda - \lambda_0}{\lambda_0} = -1.13 \times 10^{-7} \text{ cm s}^{-1} = -113 \text{ km s}^{-1} \quad (1)$$

As per convention, a negative value of V_r implies that Barnard's star is approaching earth. Note that the velocity of earth in its orbit around the sun is 30 km s^{-1} , and cannot be neglected. This is why the velocities of stars are usually reported in the barycentric frame. The refractive index of air, $n = 1.000297$ only affects the speed of light by a slight value, and can be neglected in this case as all measurements are made in air.

b) Tangential velocity The parallax of Barnard's star is $p = 0.549''$ giving a distance $d = 1.82$ pc. Then, we use the proper motion μ to calculate linear velocity:

$$v_t = d\mu = 1.82 \text{ pc} \times 10.4'' \text{ yr}^{-1} = 5.62 \times 10^{18} \text{ cm} \cdot 1.59 \times 10^{-12} \text{ rad yr}^{-1} = 89.4 \text{ km s}^{-1} \quad (2)$$

Alternately, we can use a conversion factor directly: a proper motion of $1'' \text{ yr}^{-1}$ corresponds to a linear velocity of 4.74 km s^{-1} at a distance of 1 pc. Thus, we get:

$$v_t = \frac{10.4''}{\text{yr}} \times \frac{4.74 \text{ km s}^{-1}}{1'' \text{ yr}^{-1} \text{ pc}} \times 1.82 \text{ pc} = 89.4 \text{ km s}^{-1} \quad (3)$$

c) Space velocity The total speed is simply obtained by vector adding the two perpendicular components of velocity. Thus we get $|v| = \sqrt{v_r^2 + v_t^2} = 144 \text{ km s}^{-1}$.

2 Carroll and Ostlie 7.3

a) Inclination angle An eclipse will just occur when the common tangent is along the line of sight, as shown in Figure 1. In this case,

$$\sin(90^\circ - i) = \cos i = \frac{r_1}{x} = \frac{r_2}{a - x} = \frac{r_1 + r_2}{(a - x) + x} = \frac{r_1 + r_2}{a}. \quad (4)$$

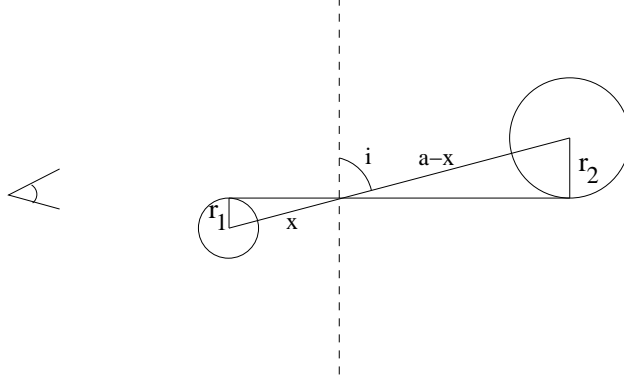


Figure 1:

The minimum i at which an eclipse occurs is then

$$i = \arccos \left(\frac{r_1 + r_2}{a} \right). \quad (5)$$

b) **Numeric example** $i = \arccos(11 R_\odot / 2 \text{ AU}) = 88.5^\circ$.

3 Carroll and Ostlie 7.6

a) **Ratio of masses**

$$\frac{m_A}{m_B} = \frac{v_{Br}}{v_{Ar}} = \frac{22.4}{5.4} = 4.14 \quad (6)$$

b) **Sum of masses** Using Eq. 7.6 in Carroll and Ostlie,

$$m_A + m_B = \frac{P}{2\pi G} \frac{(v_{Ar} + v_{Br})^3}{\sin^3 i} \quad (7)$$

$$= \frac{6.31 \text{ yr} (5.4 \text{ km/s} + 22.4 \text{ km/s})^3}{2\pi G} \frac{1}{1} \quad (8)$$

$$= 1.02 \times 10^{34} \text{ g} = 5.13 M_\odot \quad (9)$$

c) **Individual masses** If $m_A + m_B = M$ and $m_A/m_B = r$, then $m_B = M/(1 + r)$ and $m_A = M * r/(1 + r)$. In this case, $m_A = 4.12 M_\odot$ and $m_B = 1.01 M_\odot$.

d) **Radii** Assuming the orbital separation is much larger than the stellar radii, and that the orbits are circular, we can treat the velocity of the stars during eclipse as completely in the plane of the sky. For circular orbits, the maximum radial velocities given are the constant velocities throughout the orbit. The relative velocity is then $v = (5.4 + 22.4) \text{ km/s} = 27.8 \text{ km/s}$. It takes a time $t_b - t_a$ for the smaller disk to enter the larger one, so the radius of the smaller star must be

$$r_s = \frac{1}{2} v (t_b - t_a) = \frac{1}{2} (27.8 \text{ km/s}) (0.58 \text{ d}) \quad (10)$$

$$= 6.97 \times 10^{10} \text{ cm} = 1.00 R_\odot. \quad (11)$$

(Note that assuming $i \approx 90^\circ$ as we do here implies the transit is across a diameter of the large disk.) Now consider the point on the smaller disk that first eclipses the larger. From time t_a to t_c

this point traces the diameter of the larger disk (see Fig. 7.9 in Carroll and Ostlie), so the radius of the larger disk is

$$r_l = \frac{1}{2}v(t_c - t_a) = \frac{1}{2}v((t_c - t_b) + (t_b - t_a)) \quad (12)$$

$$= \frac{1}{2}(27.8 \text{ km/s})(0.64 \text{ d} + 0.58 \text{ d}) \quad (13)$$

$$= 1.47 \times 10^{11} \text{ cm} = 2.11 R_\odot. \quad (14)$$

e) Effective temperatures Following Examine 7.3.2 in Carroll and Ostlie, the ratio of fluxes between the primary minimum and maximum light is $B_p/B_0 = 100^{(5.40-9.20)/5} = 0.030$, and the ratio of fluxes between the secondary minimum and maximum light is $B_s/B_0 = 100^{(5.40-5.44)/5} = 0.964$. Then

$$\frac{T_s}{T_l} = \left(\frac{1 - B_p/B_0}{1 - B_s/B_0} \right)^{1/4} = \left(\frac{1 - 0.030}{1 - 0.964} \right)^{1/4} = 2.3. \quad (15)$$

The derivation in the text assumes that the smaller star is hotter, i.e. that the primary eclipse is when the smaller star passes behind the larger. Can we back this up with the data? Assuming this is true, then in the primary eclipse we see only the larger star, which gives $100^{(m_0-m_p)/5} = 100^{(5.40-9.20)/5} = 3.02\%$ of the total brightness. In the secondary, we then expect to see the remaining 96.98% of the flux from the smaller star, plus the fraction $(r_s/r_l)^2 = 0.225$ (from part c) of the big star's 3.02% that is unobstructed, for a total of 97.7%. We observe $100^{(m_0-m_s)/5} = 100^{(5.40-5.44)/5} = 96.38\%$, which is close considering we only two significant digits in the times.

If we instead imagine that the larger star is hotter, so that the primary eclipse occurs when the smaller star is in front, we find that the 96.38% of light visible in the secondary is due to the larger star, and that during the primary we expect 0.225 of this (plus the 3.62% contribution of the smaller star), which is far in excess of the 3.02% observed.

Another way to estimate the temperature ratio is to note that the primary minimum is 0.030 of the maximum brightness, so that

$$0.030(T_s^4 R_s^2 + T_l^4 R_l^2) = T_l^4 R_l^2, \quad (16)$$

which implies

$$\frac{T_s}{T_l} = \left[\left(\frac{1}{0.030} - 1 \right) \left(\frac{R_l}{R_s} \right)^2 \right]^{1/4} = 3.5, \quad (17)$$

using the radii from (d). This does not agree particularly well with the 2.3 we found above, indicating that the timing data in this problem are not consistent with the photometry under the assumption that the smaller star passes directly across a diameter of the larger star's disk. For this homework either answer is acceptable.

4 Carroll and Ostlie 7.11

For simplicity, assume circular orbits. (In any case, eccentricity can be deduced from distortions of the radial velocity curve from sinusoidal.) Denoting the masses of the planet and star by M_p and M_s , Kepler's third law says

$$P^2 = \frac{4\pi^2}{G(M_p + M_s)} a^3 \approx \frac{4\pi^2}{GM_s} a^3, \quad (18)$$

under the approximation that $M_p \ll M_s$, so

$$a \approx \left(\frac{GM_s P^2}{4\pi^2} \right)^{1/3}. \quad (19)$$

The orbital velocity of the planet is approximately

$$v_p \approx \frac{2\pi a}{P} \approx \left(\frac{2\pi GM_s}{P} \right)^{1/3}, \quad (20)$$

and that of the star is reduced approximately by the mass ratio:

$$v_s \approx \frac{M_p}{M_s} v_p. \quad (21)$$

We can measure the period P and estimate M_s by determining its spectral type with our spectrum. Thus, we know v_p . If we could measure v_s , then Equation 21 would allow us to solve for M_p . However, what we measure is $v_{rs} = v_s \sin i$, the radial component of the star's velocity, and the orbital inclination i is in general unknown. Since v_{rs} is a lower limit on v_s , we get a lower limit on $M_p = M_s v_s / v_p$, but cannot say more.

5 Carroll and Ostlie 7.13

Under the assumptions of a uniform solar disk and no luminosity from Jupiter, the decrease is just given by the fraction of the solar disk blocked by Jupiter:

$$\frac{\pi R_J^2}{\pi R_\odot^2} = \left(\frac{R_J}{R_\odot} \right)^2 = \left(\frac{7.1 \times 10^9 \text{ cm}}{7.0 \times 10^{10} \text{ cm}} \right)^2 = 1.1\%. \quad (22)$$