Basic Astronomy and the Galaxy

Problem Set 2

Due **in class** Monday 20 October, 2008

Homework Problems:

1. **Angular resolutions of radio and other telescopes**

   Calculate, in arcseconds, the angular resolutions of some famous diffraction-limited telescopes:

   a) Caltech's 40 m single-dish radio telescope in Owens Valley, operating at the 1.4 GHz frequency of the neutral hydrogen transition.

   b) Caltech's CARMA array of mm-wave telescopes, with longest baseline of 2 km, operating at the 115 GHz frequency of the ground-state transition of Carbon monoxide.

   c) The Arecibo single-dish radio telescope (1000-foot diameter) in Puerto Rico, operating at 1.4 GHz frequency.

   d) The national Very Large Array of radio telescopes in New Mexico, with longest baseline of 36 km, operating at 1.4 GHz frequency.

   e) The national Very Long Baseline Array, whose radio telescopes cover the continental US, with additional stations in Hawaii and the Virgin Islands, operating at 1.4 GHz frequency.

   f) The 2.4 m Hubble Space Telescope, observing at the shortest wavelength of its late lamented ACS camera, 380 nm.

   g) The 0.85 m Spitzer Space Telescope, operating at the shortest wavelength of its imaging photometer, 24μm.

   h) The planned Caltech TMT (Thirty Meter Telescope), with the adaptive optics instrument NFIRAOS (high-speed deformable mirrors and six laser-created guide 'stars' in the upper atmosphere) cancelling earth's seeing at the shortest well-correctable wavelength, 1.0 microns.

   i) NASA's planned James Webb Space Telescope at its shortest optimised wavelength, 1.2 microns.

2. **What we'd like to see** Calculate the angular sizes of[1]

   a) A sun-like star at 100pc distance from earth.

   b) A red-giant star with radius 1000 times the sun's radius, at 100pc distance from earth.

   c) The maximum separation of a pair of sun-like stars orbiting each other with a 5-day orbital period, at 100pc distance from earth.

   d) The maximum separation of a pair of sun-like stars orbiting each other with a 100-year orbital period, at 100pc distance from earth.

   e) The shell of a supernova (exploding at 10^4km s^{-1}), at 10 Mpc distance,one month after the explosion.

[1] 1kpc = 10^3pc, 1Mpc =10^6pc, 1Gpc=10^9pc

f) The region of a galaxy at 100 Mpc distance around a $10^8 M_\odot$ black hole where the stars have orbital velocities greater than 500km s^{-1} (i.e. much greater than found elsewhere in galaxies).

g) A newly formed cluster of stars (diameter 2 pc) in a pair of merging galaxies at 1 Gpc distance.

3. **Fading into the distance**

a) The faintest objects the Palomar 60-inch telescope can detect in imaging with reasonable (several minute) exposures is a Johnson-Vega magnitude $m_V = 22$. Out to what distance could it detect (i) A star like the sun, (ii) a white dwarf with the same surface temperature as the sun (white dwarf radii are 0.01 of the sun's radius), (iii) a quasar with luminosity $\nu L_\nu = \lambda L_\lambda = 10^{46}$erg s^{-1} at all optical and ultra-violet wavelengths?

b) The heroic Hubble Ultra-Deep Field imaged a single patch of sky continuously for 400 orbits, of which 56 (135,320 seconds) were in a 'V'-like band (606nm). The faintest detected sources in that 'V' band image had an *AB magnitude* (BEWARE! not Johnson Vega scale!) at 606 nm of about 27.5. At what distance (in pc) would such a heroic exposure be able to detect (i) A star like the sun, (ii) a white dwarf with the same surface temperature as the sun (white dwarf radii are 0.01 of the sun's radius), (iii) a quasar with luminosity $\nu L_\nu = \lambda L_\lambda = 10^{46}$erg s^{-1} at all optical and ultra-violet wavelengths?

4. **Spectroscopy is a lot slower than imaging**

a) Compute the specific flux F_ν (in Jy, and erg/cm2/s/Hz) and F_λ in erg cm^{-2}s^{-1}Å^{-1} from a V = 22m galaxy. What is the rate at which V-band photons from this object in the V band strike the mirror of the Hale 200-inch-diamter telescope from this object?

b) A spectrum of this galaxy is taken. The effective entrance aperture of the spectrograph is 2×2arcsec^2. The surface brightness of the night sky at Palomar in the V band on a dark night is 20.4 mag/arcsec2 (i.e., 1 arcsec2 emits a flux equivalent to that of a V = 20.4 mag object). What is the effective V magnitude of the foreground sky patch as seen by the spectrograph in its aperture?

c) Now assume that the overall efficiency of the telescope + spectrograph + detector is 10%. A resolution element in the spectrum is 10Åwide, and we are interested in the region around $\lambda = 5500$Å. How many counts per resolution element are detected from the galaxy spectrum alone in a 1-hour exposure? From the foreground sky?

d) If a blank piece of sky is measured at the same time in order to subtract the sky spectrum from the total, what is the signal-to-noise ratio per resolution element in the final, sky-subtracted galaxy spectrum? (Neglect the detector noise, and assume Poisson photon statistics.)

5. **Zodiacal light and the search for extrasolar earth-like planets**

Dust released into the solar system from asteroid collisions and comets slowly spirals into the sun. It was first discovered because it scatters sunlight, creating a 'glow' in the dark night sky.

a) Astronomers quote the surface brightness of the zodiacal light in the anti-solar direction as seen from earth as 60 S_{10}'s. S_{10} is an awful unit, equal to the (V-band) surface brightness that one would get by spreading the light from a star of $m_V = 10$ over 1 square degree. Calculate the flux of zodiacal light from the dark night sky, and thus compute the fraction of the sun's light scattered by the zodiacal dust located beyond the earth's orbit.

b) What fraction of the sun's light is scattered by the earth?

c) At a distance of 10 pc, what magnitude would be an earth-like planet in a 1AU orbit around a sun-like star, when it is at its largest elongation from its star?

d) The zodiacal light is observed to have about the same optical colors as sunlight, which implies that the dust grains must be larger than optical wavelengths, i.e. their size $a \gtrsim 1\mu$m. What is an approximate lower limit to the ratio of the total mass of zodical dust particles to the mass of the earth (assume that most of the zodical dust particles are within 2 AU of earth)?

e) Suppose we observe with a 10 m diameter space telescope, in green light, a sun-like star, 10 pc distant, orbited by a solar system identical to ours. What is the ratio of the flux from that solar system's zodiacal light in the resolution element containing its earth, to the actual flux from its earth, when its earth is at the largest possible angle from the star?

f) Scattered light from the star is clearly a further problem for the observation in the previous problem. So it has been proposed to search instead in the infrared. Assume that half the sunlight hitting the zodiacal dust grains is absorbed and reradiated as a black body (and the other half scattered, producing the zodical light).

   i. What is the wavelength λ_p at which this blackbody reradiation from zodical dust has a peak in λB_λ (flux per log wavelength interval)?

   ii. Assuming that the sun radiates as a 5777 K black body at $\lambda > \lambda_p$, what would be the ratio of the flux from the star to the reradiated flux from the zodiacal light at $\lambda \gg \lambda_p$?

   iii. at $\lambda \gg \lambda_p$, what would be the ratio of the flux from the earth-like planet to the reradiated flux from the zodiacal light?

   iv. Operating at wavelength λ_p, what aperture telescope would be required to clearly separate the earth-like planet from the star's image?

You have now completed the design trade study for NASA's Terrestrial Planet Finder mission.