Ay 20 Basic Astronomy and the Galaxy
Problem Set 1

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1 What's up?

We begin with the Vernal Equinox on 2008 March 19 (PST), when the sun has a right ascension (RA) of 0^h. (The equinox is more precisely at 05:48 UT on March 20, or 21:48 PST on March 19, but we will neglect the small correction due to the exact time of the equinox, which is well within the "error bars" of this problem.) So at midnight, when the sun is directly below us, the RA on the meridian is 12^h. Since 2008 November 1 is 227 days later, and sidereal time advances relative to solar time at 3 min 56 sec per day, the RA on the meridian at midnight is $12^h + 227 \times 3^m 56^s \pmod{24} = 2^h 53^m$.

Now the RA on the meridian increases by about an hour for every hour of UT time (the small difference between 4 sidereal and 4 solar hours is negligible here), so the RA on the meridian at 8 pm is 4 hours lower (mod 24) than at midnight, i.e. $\boxed{22^h 53^m}$. Similarly, the RA on the meridian at 4 am is 4 hours larger than at local midnight: $\boxed{6^h 53^m}$. This is a fully acceptable answer to the problem.

Now we dive into the complications caused by the timezones. For a map of timezones see http://aa.usno.navy.mil/graphics/TimeZoneMap2007.pdf The longitude of Palomar is $116°52'$ west. Time zones are centered on longitudes that are a multiple of 15°, so the relevant timezone (in this case PST) is centered on 120° west. The difference in longitudes is 3.13°. Since a difference of 15° corresponds to 1 hour, 3.13° corresponds to 13 minutes. Hence when it is 8 pm PST on 1 November, or 8 pm locally at the 120° longitude, the local time at Palomar will be 8:13 pm, and the RA at meridian will be $\boxed{23^h 06^m}$. Similarly, at 4 am, the RA at the meridian will be $\boxed{7^h 06^m}$.

In reality, the local times will be somewhat less than this: $22^h 59^m$ and $6^h 59^m$. The main difference arises because local noon at Greenwich does not correspond exactly to 0^h UT on the vernal equinox, as was assumed. UT is designed so that this correspondence holds on the *average*, but it will not be precisely true at the equinox. Sidereal time is in fact non-uniform, due to the eccentricity of Earth's orbit, its axial tilt, precession, and other effects, making sidereal time drift seasonally with respect to UT. For more precise work, see the *Astronomical Almanac* published annually by the US and British naval observatories.

Finally, note that we do not revert from daylight saving time (PDT) to standard time (PST) until 2 am PDT on 2008 November 2. Thus, PST is not the time on our watches for part of the period of this problem!

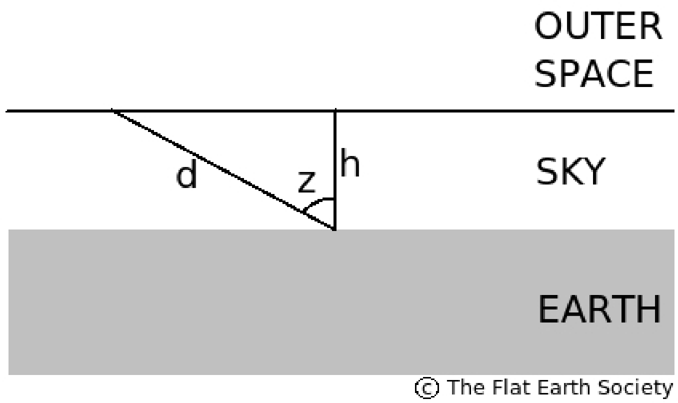


Figure 1: A simplified calculation for airmass

2 What's feasible?

Let us first derive the relation of airmass with the zenith angle z. For this we use the toy model of the sky shown in Figure 1. We assume a simple, plane parallel atmosphere, so the amount of air that incoming starlight passes through is directly proportional to the length d of the air column it traverses (notice that this is true even if the atmosphere is not uniform in density, as long as the iso-density contours are horizontal, since in each thin layer, the contribution to the column density varies as $\sec z$). If we assume the atmosphere to have a height h, then for a zenith angle z we see that $d = h \times \cos(z)$. While looking directly at the zenith, the airmass is defined to be 1. Thus we get,

$$\text{airmass} = \frac{d}{h} = \frac{1}{\cos(z)} = \sec z. \tag{1}$$

For an airmass of 2.5, the zenith angle is $z = 66.4°$, or $66°25'$. The latitude of Palomar is $33°21'$, so the declination of zenith is also $33°21'$. The extreme declinations we can see are thus $33°21' \pm 66°25'$. The minimum declination is $\boxed{-33°4'}$. The maximum declination given by this formula is $99°46'$ which just means that the airmass for the North Celestial Pole is less than 2.5 and we can dip our telescope even further down. This corresponds to a different right ascension. The maximum declination is just $\boxed{90°}$.

3 α Centauri System

(a) Angular Separation Looking at the coordinates, we see that the stars are close enough that we can use a planar approximation to solve the problem. The differences in right ascension and declination respectively are $\Delta\alpha = 0^h 9^m 53^s$ and $\Delta\delta = 1°50'38''$. Before applying the textbook formula, note that the right ascension needs to be converted to degrees! The formula thus becomes,

$$\Delta\theta = \sqrt{(\Delta\alpha \cdot 15\cos\delta)^2 + (\Delta\delta)^2}. \tag{2}$$

So we get $\boxed{\Delta\theta = 2.2°}$. Several of you neglected the $\cos\delta$ factor here. Recall that RA differences do not directly measure angles on the sky (that is, as measured along the arc of a great cricle). Think of two points on just opposite sides of the celestial north pole. They have $\Delta\alpha = 12^h = 180°$, but can be at an arbitrarily small angular separation.

(b) Physical Distance From the parallaxes, we can calculate the distances:

$$d = \frac{1}{\theta_{parallax}}, \tag{3}$$

where d is in parsecs and θ in arcseconds. The distance to Centauri A is thus 1.35 parsecs (pc), and that to Proxima is 1.32 pc. The physical distance is then given by,

$$r = \sqrt{(\Delta d)^2 + (d \cdot \Delta\theta)^2}. \tag{4}$$

Thus the physical separation in the two stars is 0.06 pc. (Which is also 12000 AU, 1.8×10^{17} cm, 0.19 light years...let's keep our answers to pc or AU!)

(c) Angular Diameter The sun subtends $32'$ at 1 AU. The distance to Centauri A is 1.35 pc. Since there are 206265 AU per pc, this is 2.78×10^5 AU. (The conversion factor simply comes from geometry: it is the number of arcseconds in a radian.) The angular size of an object with the same physical size as the sun is then $32' \times \frac{1}{2.78 \times 10^5} = 1.15 \times 10^{-4}$ arcmin $= 0.0069$ arcsec. (The resolution of Hubble is only 0.1 arcsec, for comparison!)

4 Sun as an (approximate) black body

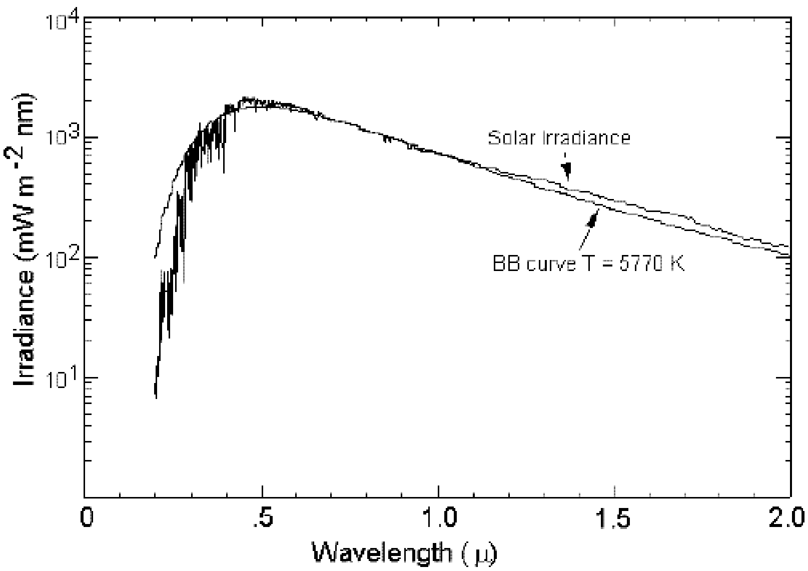
(a) Integrate We have to calculate the integrated intensity of a blackbody with a temperature $T_\odot$=5777 K. (Note that the symbol $\odot$ always refers to the sun, and $\oplus$ refers to earth.) The specific intensity of a blackbody is given by

$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}. \tag{5}$$

For this problem set, it is sufficient to put the numbers in your favourite math package and get the numerical answer. Setting $T = T_\odot$=5777 K and integrating from $\lambda = 0$ to ∞ gives

$$\int_0^\infty B_\lambda(T_\odot) d\lambda = 2.0 \times 10^{10}\,\mathrm{erg\,cm^{-2}s^{-1}sr^{-1}} = 2.0 \times 10^7\,\mathrm{J\,m^{-2}s^{-1}sr^{-1}}. \tag{6}$$

Just as a matter of interest, the figure below shows the actual solar irradiance compared to that of a black body at the effective temperature of the sun (i.e., the black body with the same total energy output as the sun). Looks pretty close, but beware of the log scale!



(b) Stephan–Boltzmann radiation constant The Stephan–Boltzmann radiation constant is $\sigma = 5.67 \times 10^{-5}\,\mathrm{erg\,cm^{-2}\,s^{-1}} = 5.67 \times 10^{-8}\,\mathrm{J\,m^{-2}\,s^{-1}}$. Putting in the numbers we get,

$$\frac{\sigma T^4}{\pi} = 2.0 \times 10^{10}\,\mathrm{erg\,cm^{-2}s^{-1}sr^{-1}} = 2.0 \times 10^{7}\,\mathrm{J\,m^{-2}s^{-1}sr^{-1}}, \tag{7}$$

which is numerically equal to the value we obtained in Equation 6.

(c) Photodiode band The `S2386-18K` photodiodes are sensitive in the wavelength range 300 nm$<\lambda<$1100 nm. The flux emitted in this range by a blackbody at $T_\odot$ is

$$\int_{300\,nm}^{1100\,nm} B_\lambda(T_\odot)d\lambda = 1.46 \times 10^{10}\,\mathrm{erg\,cm^{-2}s^{-1}sr^{-1}} = 1.46 \times 10^{7}\,\mathrm{J\,m^{-2}s^{-1}sr^{-1}}. \tag{8}$$

Thus, 73% of the sun's energy comes out in the range of wavelengths to which the photodiode is sensitive.

5 Flux from the sun in a clever way

(a) Solid angle of the sun Recall that an angle is the ratio of the length of an arc to the radius. Similarly, solid angle is the ratio of area on a sphere's surface the square of the sphere's radius. Let us denote the radius of the sun by $R_\odot$ and the earth–sun distance by d. The solid angle is given by

$$\Omega_\odot = \frac{\text{Projected area of sun}}{(\text{Earth sun distance})^2} = \frac{\pi R_\odot^2}{d^2}. \tag{9}$$

But, we know that $\theta = \frac{2R_\odot}{d}$ is the angular diameter of the sun, 32' (0.0093 radians). Thus,

$$\Omega_\odot = \pi \cdot \left(\frac{\theta}{2}\right)^2 = 6.8 \times 10^{-5}\,\text{steradians} \tag{10}$$

(b) Flux from solid angle In part 4(b) we showed that the energy emitted by the sun per unit area per unit time per unit solid angle is given by I = $\sigma T^4/\pi$. To obtain the flux at the solar surface, we integrate $I\cos\theta$ over all outgoing directions (i.e., θ ranging from 0 to $\pi/2$), assuming I is the same in all such directions. Since the differential solid angle element is $\sin(\theta)d\theta d\phi$, we get

$$F = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2} I\cos(\theta)\cdot\sin(\theta)d\theta d\phi = 2\pi I\int_{\theta=0}^{\pi/2}\cos(\theta)\sin(\theta)d\theta = \pi I. \tag{11}$$

This is nothing but the Stephan–Boltzmann radiation law, $F = \sigma T^4$. The total luminosity of the sun is then given by the product of F and its surface area. The flux at earth $F_\oplus$ is obtained by spreading this total power over an area of $4\pi d^2$, where d is the earth–sun distance:

$$F_\oplus = \frac{L_\odot}{4\pi d^2} = \frac{F\cdot 4\pi R_\odot^2}{4\pi d^2}. \tag{12}$$

Combining with Equation 9 and substituting $F = \pi I$ we get $F_\oplus = I(T_\odot)\Omega_\odot$. Numerically, $F_\oplus = 1360\,\mathrm{W\,m^{-2}} = 1.36\times10^6\,\mathrm{erg\,cm^{-2}\,s^{-1}}$.

Notes: Where does the $\cos\theta$ factor come from? Recall that in computing flux, we count only the component normal to the surface. One way to understand this is the following: in computing flux, we want the energy passing outward through an area dA_1 of the stellar surface per unit time, regardless of its direction. The dA_2 in intensity, however, is measured perpendicular to the outgoing ray, and the stellar patch dA_1 has a projected area $dA_2 = dA_1\cos\theta$ in this direction. We thus need to multiply I by $\cos\theta$ to reference the area to that of the stellar surface.

Many of you had trouble with this π factor. The *intensity* is measured *per solid angle*. To convert the intensity to the flux at the stellar surface, we have to integrate over solid angle to obtain $F = \pi I$. (The flux at earth, of course, is then diminished by distance in the usual way.)

(c) Flux from luminosity For this calculation we use the radius of the sun $R_\odot = 6.96\times10^8$ m, earth–sun distance $a = 1.5\times10^{11}$ m, $\sigma = 5.67\times10^{-8}\,\mathrm{W\,m^{-2}}$ and $T_\odot = 5777$K. Thus, we get $L_\odot = 3.84\times10^{26}$ W. The flux at earth is then F = $L_\odot/(4\pi a^2) = 1360\,\mathrm{W\,m^{-2}}$, the same as in part (b).

6 Photon counts from a blackbody

(a) Photons counts The energy emitted by a blackbody in a wavelength range dλ centered on the wavelength λ is $B_\lambda d\lambda$. Hence the number of photons in this range is obtained by dividing this value by the energy of a photon, hc/λ. Put the integral in your favourite numerical package or your own code to get the answer:

$$\int_{300\mathrm{nm}}^{1100\mathrm{nm}}\frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 4.8\times10^{21}\ \text{photons cm}^{-2}\ \text{s}^{-1}\ \text{sr}^{-1} \tag{13}$$

(b) Photons in V–B–U bands Since we are assuming 100% transmission in the bands and 0% outside, this is simply the same integral as Equation 13, with different limits:

$$n_V = \int_{505\mathrm{nm}}^{595\mathrm{nm}}\frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 6.4\times10^{20}\ \text{photons cm}^{-2}\ \text{s}^{-1}\ \text{sr}^{-1} \tag{14}$$

$$n_B = \int_{391\text{nm}}^{489\text{nm}} \frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 5.4\times 10^{20} \text{ photons cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1} \tag{15}$$

$$n_U = \int_{331\text{nm}}^{399\text{nm}} \frac{B_\lambda(5777\,K)d\lambda}{hc/\lambda} = 2.5\times 10^{20} \text{ photons cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1} \tag{16}$$

7 Palomar observing

(a) Specific flux Magnitudes are defined such that

$$m_2 - m_1 = -2.5\log\frac{F_2}{F_1}, \tag{17}$$

so if we know the flux and magnitude of a single object, we can compute the flux corresponding to any other magnitude:

$$F_2 = F_1 10^{-0.4(m_2-m_1)}. \tag{18}$$

A convenient reference object is the sun, for which we can use the calculations developed in this problem set.

Approximating the sun as a blackbody, we can compute its flux on Earth in the V band using the same technique as in problem 5, now using the V band for our integration limits. As in problem 6, we (crudely) regard the V filter as extending from 505nm to 595nm with perfect transmission, and having no transmission outside this interval.

$$I_{\odot V} = \int_{505\text{nm}}^{595\text{nm}} B_\lambda(5777\text{K})d\lambda = \int_{505\text{nm}}^{595\text{nm}} \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} = 2.3\times 10^9 \text{ erg cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1} \tag{19}$$

The flux at Earth is then $F_{\odot V} = I_{\odot V}\Omega_\odot = (2.3\times 10^9 \text{ erg cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1})(6.8\times 10^{-5}\text{ sr}) = 1.6\times 10^5 \text{ erg cm}^{-2}\text{ s}^{-1}$.

From the inner cover of Carroll and Ostlie, the apparent magnitude of the sun in V band is $m_{\odot V} = -26.75$. A star with $m_V = 14$ therefore has a V band flux of $F_V = F_{\odot V}10^{-0.4(14-m_{\odot V})} = 7.9\times 10^{-12} \text{ erg cm}^{-2}\text{ s}^{-1}$. To get the flux per wavelength, we divide by the bandpass $\Delta\lambda = 90$ nm and obtain $\boxed{F_\lambda = 8.7\times 10^{-14} \text{ erg cm}^{-2}\text{ s}^{-1}\text{ nm}^{-1}}$ as an average in the V band. (Nothing more precise can be said with broadband photometry alone.)

Similar calculations for a $m_V = 21$ star give $F = 1.2\times 10^{-14}$ erg cm^{-2} s^{-1} and $\boxed{F_\lambda = 1.4\times 10^{-16} \text{ erg cm}^{-2}\text{ s}^{-1}\text{ nm}^{-1}}$.

(b) Photon counts We make the approximation that all photons in the V band have the energy corresponding to the central wavelength of 550nm: $E_\gamma = hc/(550\text{nm}) = 3.6\times 10^{-12}$ erg. (Note: To go beyond this approximation, we would need to know the spectrum of the object.) The number of photons collected by P60 per second is then $F/E_\gamma \times \pi r^2$, where $r = 30$ inches for P60. Using the fluxes from part (a), this is $\boxed{4.0\times 10^4 \text{ s}^{-1}}$ for the $m_V = 14$ star and $\boxed{60 \text{ s}^{-1}}$ for the $m_V = 21$ star. *Note*: Since no efficiency information was given in the problem, this calculation assumes that 100% of photons incident at the top of the atmosphere are counted by the CCD, which is of course false. CCD 13 at P60, the one most commonly used for photometry, has a quantum efficiency of about 63% in the middle of the V band. See http://www.astro.caltech.edu/palomar/ccds/QE.gif. (CCD 9 is only available for the echelle spectrograph.) You will want to account for this in your observing proposals. Additionally, some light is absorbed by the Earth's atmosphere, and this depends on the object's airmass and color.

(c) Sky background The sky brightness in V band is given as 20.4 mag arcsec^{-2}. Using the same calculation as part (a), the flux for a 20.4 mag source is $F = F_{\odot V} 10^{-0.4(20.4 - m_{\odot V})} = 2.2 \times 10^{-14}$ erg cm^{-2} s^{-1}. We convert this to a photon arrival rate at P60 as in part (b): $F/E_\gamma \times \pi r^2 = 110$ s^{-1}. This is the photon rate for 1 arcsec^2 of sky. Since the seeing disk has diameter 1.5 arcsec, its area is $\pi(\frac{1}{2}1.5 \text{ arcsec})^2 = 1.77$ arcsec^2. The rate at which P60 receives photons from the whole seeing disk is thus $\boxed{1.77 \times 110 \text{ s}^{-1} = 194 \text{ s}^{-1}}$. (The same comments about efficiency from part (b) apply here.)

(d) Photometric precision In a 60 s exposure, 60 s $\times$ 60 s^{-1} = 3600 photons are received from the $m_V = 21$ star, and 60 s $\times$ 194 s^{-1} = 11640 are received from the sky. The Poisson error in the total flux is thus $\sqrt{3600 + 11640} = 123$. An acceptable answer is that the error is thus $123/3600 = \boxed{3.4\%}$.

Several of you noted that there is an additional error that arises from sky subtraction, and that since $N_{\text{object}} = N_{\text{object+sky}} - N_{\text{sky}}$, the errors add as $\sigma_{\text{object}} = \sqrt{\sigma^2_{\text{object+sky}} + \sigma^2_{\text{sky}}}$. This is correct. However, it's hard to say what σ_{sky} is, since we aren't told the number of pixels over which the sky is measured (which is typically much larger than the number in a single seeing element). Also, note that this calculation is a lower limit, since we have neglected other sources of noise, such as readout noise and flat-fielding errors. Finally, inclusion of the proper efficiency factor f as discussed in part (b) will affect the S/N calculation, since the noise scales as $\sqrt{f}$ and the signal as f.

(e) Saturation In part (b) we found that 4.0×10^4 photons were collected per second for an $m_V = 14$ star. (This is so much larger than the sky background that we can ignore it; however, for a faint object, the background will dominate this calculation.) These are distribution over an area of 1.77 arcsec^2 (part (c)), or about 12 pixels. We can thus integrate for $10^5/(4.0 \times 10^4 \text{ s}^{-1}/12) = \boxed{30 \text{ s}}$ before the pixels saturate. (In reality, the light is not spread uniformly over a disk, but is instead peaked in the center, which has to be accounted for in saturation calculations.)