Problem Set 1

Due in class Monday 6 October, 2008

Readings: To complete this problem set, and your observing proposal (due Mon Oct 13) you will need to have read chapters 1, 3 and 6 of Carroll & Ostlie.

Homework Problems:

Palomar observatory is at latitude 33° 21’ 21” N, longitude 116° 51’ 50”.

1. What’s up? The night of November 1-2, 2007 is in the midst of the class observing run. Celestial objects in what range of right ascension (RA) cross the meridian at Palomar during that night, i.e. between 8pm PST (Pacific Standard Time) and 4am PST? Your answer need not be more accurate than 15m in RA.

2. What’s feasible? Celestial objects in what range of declinations ever attain airmass less than 2.5 as observed from Palomar (airmass is defined as 1 at the zenith, and is proportional to the total column of atmosphere traversed. You may assume a plane parallel atmosphere, for which you should show that airmass = sec z, where z is the zenith angle)? Your answer need not be more precise than half a degree in dec.

3. α Centauri System. Proxima Centauri is the closest star to the Sun and is a part of a triple star system (the other two are called α Centauri A and B). It has epoch 2000.0 coordinates (\( \alpha, \delta \)) = (14h 29m 43s, –62° 40’ 46”) while the brightest star in the system, α Centauri A, is located at (\( \alpha, \delta \)) = (14h 39m 36s , –60° 50’ 8”).
   
   a) What is the angular separation between Proxima Centauri and Centauri A?
   
   b) If the trigonometric parallaxes for Proxima Centauri and Centauri A are measured to be 0.758” and 0.741”, respectively, what is the total distance between the two stars? Give your answer in all of the following units: parsecs, astronomical units, and centimeters.
   
   c) What is the angular diameter of Centauri A, assuming that its radius is similar to that of the sun? [you should be able to do this problem without looking up the radius of the sun in meters!]

4. Sun as an (approximate) black body The effective temperature of the sun is 5777K. Assume (as is only crudely true) that its spectrum is that of a blackbody with that temperature.
   
   a) Integrate \( \int_0^\infty B_\lambda(5777K) \, d\lambda \) to find the integrated intensity \( I(5777K) \) (in erg cm\(^{-2}\)s\(^{-1}\)sr\(^{-1}\) or its MKS equivalent) radiated by a black body at the temperature of the sun.
   
   b) Show numerically that \( I = \sigma T^4 / \pi \), where σ is the Stefan-Boltzmann factor.
   
   c) Silicon photodiodes like the Hamamatsu S2386-18K used in class have quantum efficiency \( \eta \sim 0.75 \) for 300nm < \( \lambda \) < 1100nm, with almost no sensitivity at longer or shorter wavelengths. Integrate \( B_\lambda(5777K) \) from 300nm to 1100nm. What fraction of the blackbody sun’s energy comes out in the range of wavelengths to which the photodiode is sensitive?
5. **Flux from the sun in a clever way**

As we showed by experiment on the first day of class, on Oct 1 the sun subtends an angular diameter of 32 arcmin.

a) Calculate the solid angle $\Omega_{\odot}$ subtended by the sun, in steradians.

b) Show that the flux (in W m$^{-2}$s$^{-1}$ or its cgs equivalent) of solar radiation on earth is $F = I(5777\text{K})\Omega_{\odot}$, and calculate this value numerically. Notice that to calculate the flux from a blackbody of known temperature (or other source of known specific intensity), you do not need to know the distance or luminosity, but only the temperature and angle subtended! Both of these are direct observables, unlike distance and luminosity.

c) Show that the answer to the previous part is the same as you would get by the more obvious but unnecessarily complicated method $F = L_{\odot}/(4\pi a)^2$, with $a = 1\text{AU}$ and $L_{\odot} = 4\pi R_{\odot}^2\sigma T^4$.

6. **Photon counts from a black body**

a) Show that the number of photons of wavelength between 300nm and 1100nm emitted from a black body at the sun’s effective temperature is

$$\int_{300\text{nm}}^{1100\text{nm}} \frac{B_\lambda(5777\text{K})}{hc/\lambda} \lambda d\lambda = 4.8 \times 10^{21} \text{photons cm}^{-2}\text{s}^{-1}\text{sr}^{-1} \tag{1}$$

b) Predict also the number of photons emitted into the passbands of the V (505nm–595nm), B (391nm–489nm) and U (331nm–399nm) filters (for the purpose of this problem, you may approximate the filters as having 100% transmission in their passband, and 0% outside of it).

7. **Palomar observing** Typical seeing at Palomar is 1.5 arcsec, which for the purposes of this problem you may take to mean that atmospheric jitter smears the image of a star over a uniformly bright disk of diameter 1.5 arcsec. In V band, the surface brightness of the usable parts of the night sky at Palomar when the moon is not up is about 20.4 V-band magnitudes per square arcsecond (due mainly now to scattered light from streetlights). The 60-inch telescope at Palomar (P60) Consider two stars of $m_V = 14$ and $m_V = 21$, respectively.

a) Calculate the flux per unit wavelength $F_\lambda$ near 550 nm (center of V-band) from each star at earth, in erg cm$^{-2}$s$^{-1}$nm$^{-1}$.

b) Calculate the number of photons collected per second in the V-band by the 60-inch telescope for each star.

c) Calculate the number of photons per second in the V-band collected from the sky in the 1.5 arcsec seeing disk.

d) How accurately can you determine the flux of the $m_V = 21$ star in a 60-second exposure? (remember that you can subtract background, but that photon arrival is a Poisson random process, so the fluctuations about a mean of $N$ photons are about $\sqrt{N}$).

e) The CCD you will use at the P60 has a pixel size of 0.378 arcsec on a side. How long an exposure can you take of the $m_V = 14$ star before you have accumulated more than $10^5$ photons per pixel, saturating the pixels electronic well?