Astronomy 127

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Cosmology and Galaxy formation

11 May 2007

TA office hours for this problem set

Dan Grin: Weds 5-6pm and Thurs 5-7pm. If many students turn up, Dan may move to 124 Bridge ("Interaction Room").

Readings:

Coles chapter 17 or Dodelson chapter 8.

Problem Set 7

Due in class *in class Fri, 18 May 2007*

Homework Problems:

1. **Cosmological parameters from CMB data** In the CMB temperature power spectrum C_l (see Problem Set 5, #3), there is a characteristic multipole moment l (or equivalently, $\theta \simeq \pi/l$) that separates small angular scales from large angular scales. The multipole moment is the l_{sh} corresponding to the sound horizon at the surface of last scattering at redshift $z \sim 1100$. Usually when you see plots of the CMB power spectrum, it is $l(l+1)C_l$ that is plotted. As discussed in lecture, for $l < l_{sh}$, $l(l+1)C_l$ is constant, and for $l > l_{sh}$ there are a series of acoustic peaks that result from oscillations in the photon-baryon fluid before recombination. The multipole moment l_{sh} is, roughly speaking, the value of l at which the first acoustic peak in the CMB power spectrum appears (observed at $\sim l = 200$).

   a) Calculate the angle θ_{sh} subtended by the sound horizon at the surface of last scattering for universes with a cosmological constant, which need not be flat (i.e. give equations valid for both open and closed universes). Assume for the moment that the sound speed in the baryon-photon fluid is $c/\sqrt{3}$ (we will correct this in part c).

   b) Using your results in part (a), plot contours of constant θ_{sh} in the $\Omega_\Lambda - \Omega_m$ plane, for $0 < \Omega_m < 1$ and $0 < \Omega_\Lambda < 1$, (N.B. $\Omega_\Lambda = 1 - \Omega_m$ for a flat universe).

   c) Calculate at recombination the sound speed of waves in the baryonic matter-radiation fluid (containing ions, electrons and radiation coupled to each other electromagnetically, unlike dark matter, supposed to couple only gravitationally). Use the relativistic expression for sound speed (replace the Newtonian ρ by $\rho + p/c^2$, and remember that p comes almost exclusively from the radiation, while both protons and radiation contribute to the mass-energy density ρ):

   $$c_s(z_r) = \sqrt{\frac{4}{3}\frac{p}{(\rho + p/c^2)}}$$

      i) How sensitive are your results to the value of Ω_b?

      ii) How much do they differ from the radiation-dominated sound speed of $c_s = c/\sqrt{3}$ you adopted in part (a)?

      iii) Would the first acoustic peak in the CMB power spectrum move to higher or lower l if Ω_b were increased?

d) Suppose that instead of a cosmological constant, the dark energy had $w = -0.9$. Would the first acoustic peak move to higher or lower l than if $w = -1$ (all other parameters fixed)? You may answer this by explanation rather than calculation.

2. **Mass-Temperature for Clusters of galaxies**

a) If one observes at redshift z a set of virialized objects at a radius such that the interior matter density is a fixed multiple Δ (> 178) of the critical density of the universe, show that the virial theorem predicts a relation between the virial temperature and the mass M_Δ within that radius

$$M_\Delta \propto T_{vir}^{3/2} E(z)^{-1}$$

where $E(z) = H(z)/H_0$.

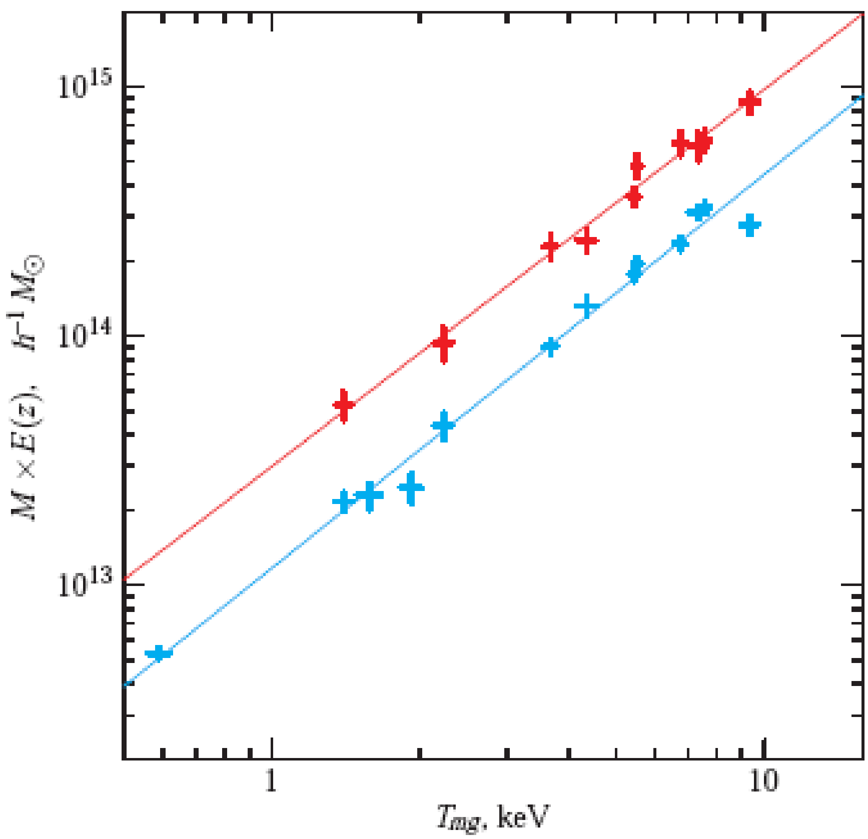

Figure 1. (Figure 19b of Vikhlinin et al astro-ph/0507092)

b) Figure 1 shows the total mass within radii where the density is 500 (top, red) times the critical density and 2500 (bottom, blue) times the critical density versus X-ray temperature, for a sample of clusters of galaxies observed by the Chandra satellite. The *slopes* of the lines are 1.51(11) and 1.58(7) respectively, in excellent agreement with the prediction of part (a). What does the *intercept* of the line tell us?

3. **Faber-Jackson and Tully-Fisher relations for galaxies** In the region of galaxies ($z = 1 - 6$ and $M \sim 10^9 - 10^{13} M_\odot$) the plot of $\sigma(R, z) \equiv \langle (\delta M/M)^2 \rangle^{1/2}$ versus R can be approximated as $\sigma(R, z) \propto R^{-s}/(1 + z)$.

a) Estimate the numerical value of s from the plot.

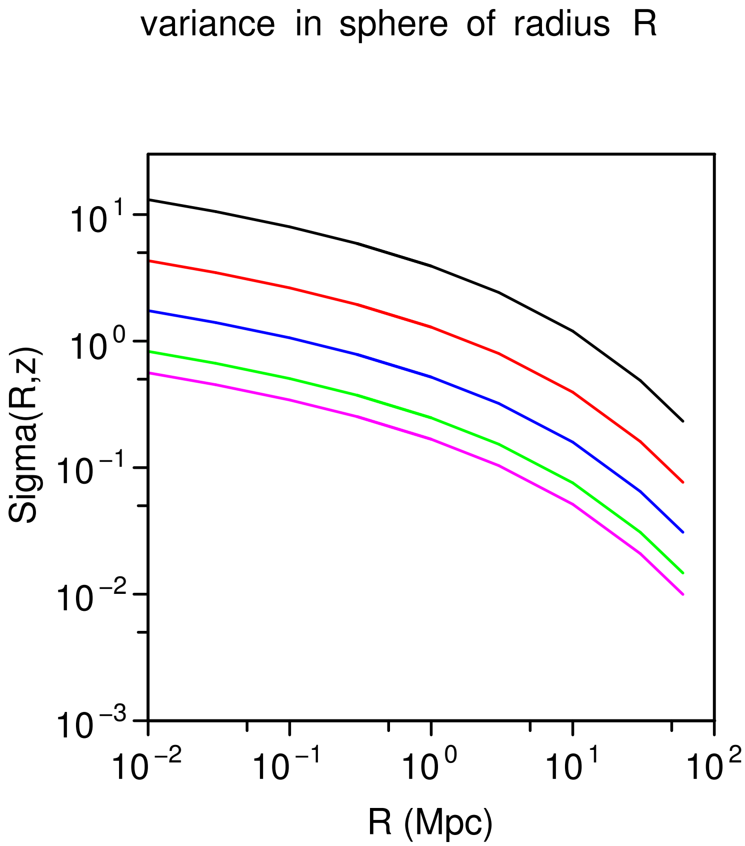

Figure 2: top hat-filtered variance $\sigma_R = \sqrt{(\delta M/M)^2} = \int_0^\infty \Delta^2(k) W^2(kR) d\ln k$ in the mass enclosed within comoving radius R, for redshifts (top to bottom) 0,3,10,20,30.

b) Show that if galaxies are virialized ν-sigma perturbations (i.e. they collapsed when their linearly extrapolated overdensity was $1.68/\nu$, with $\nu \sim 1-3$) that

i) the collapse redshift z_c is given by $(1+z_c) \propto M^{-s/3}\nu$

ii) the mass and circular velocity are related by

$$M \propto v_c^{6/(2-s)} \nu^{-3/(2-s)} \; .$$

Insert your numerical value of s into this $M(v_c, \nu)$ scaling, and compare to the observed data of Fig 3.

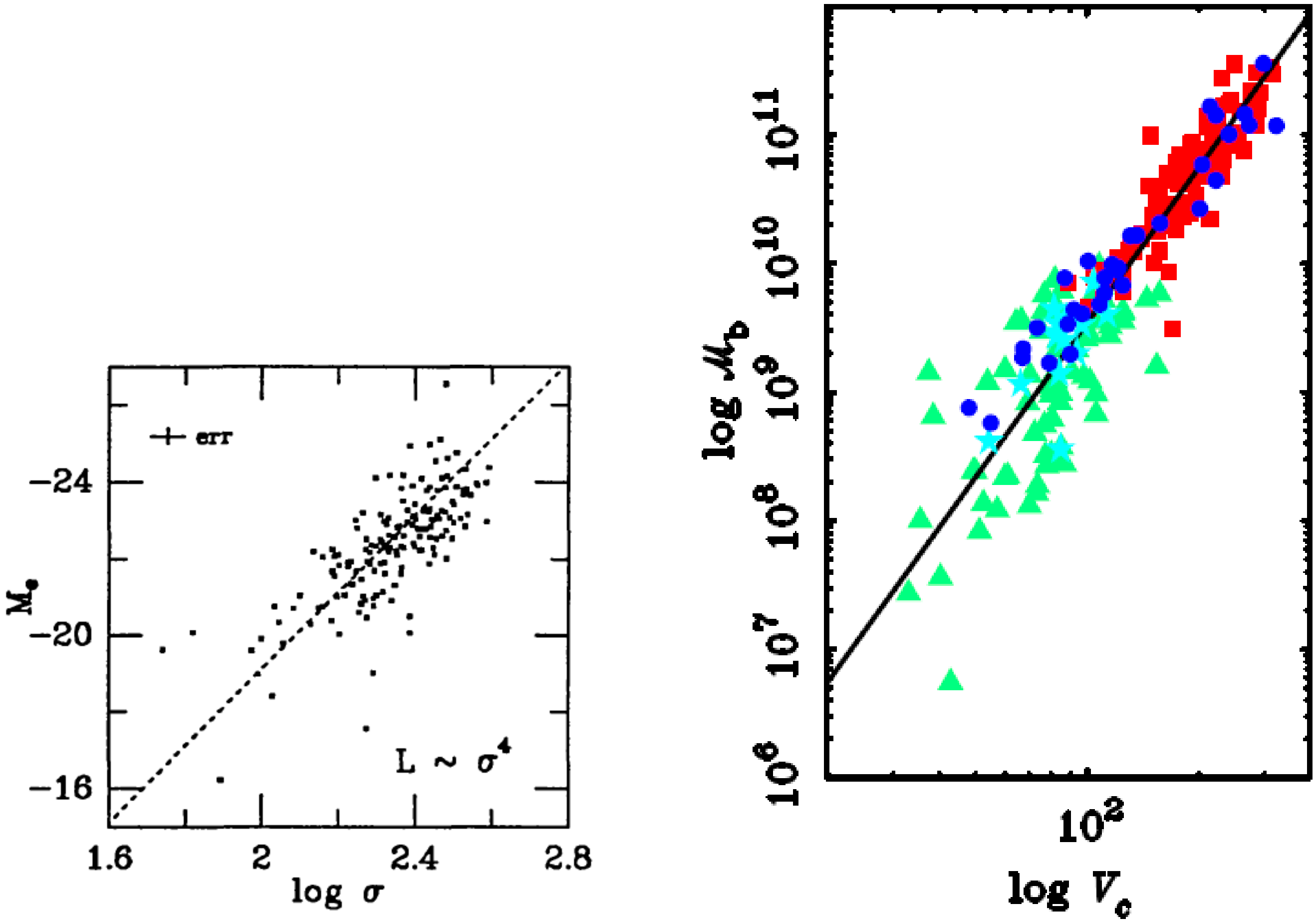

Figure 3. [left panel] Faber-Jackson relation: Magnitude *NOT mass!!* versus velocity dispersion $\sigma \sim v_c/\sqrt{2}$ of elliptical galaxies, from Kormendy & Djorgovski 1989 ARAA 27, 235, fig2. [right panel] Tully-Fisher relation: Total baryon mass (stars plus gas) vs rotation velocity for disk ('spiral') galaxies, from McGaugh 1999 astro-ph/9909452, fig 1.

c) Elliptical galaxies of a given mass have a higher v_c than spiral galaxies of the same mass. In your simple model of part (b), do you then expect ellipticals to have larger or smaller ν than spirals? What consequence would this have for their predicted clustering?

4. **Why do we expect the CMB to be polarized?** In this problem, you will show that temperature anisotropies in the CMB generate polarization. Specifically, the light Thomson scattered off an electron is polarized if and only if the radiation that is incident on the electron has a quadrupole temperature anisotropy.

Consider a single ray of light that is scattered in the z-direction by an electron at the origin. The scattered light has two independent polarization vectors $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$. The incident light ray originates from some direction $\hat{n}$ and has two independent polarization vectors $\hat{\tilde{\varepsilon}}_1(\hat{n})$ and $\hat{\tilde{\varepsilon}}_2(\hat{n})$. The angular cross section for Thomson scattering light that is polarized in the $\hat{\varepsilon}_i$ direction is

$$\frac{d\sigma_i}{d\Omega} = \frac{3\sigma_T}{16\pi} \sum_{j=1}^{2} \left| \hat{\varepsilon}_i \cdot \hat{\tilde{\varepsilon}}_j(\hat{n}) \right|^2$$

where σ_T is the Thomson cross section. Now replace the single electron by an electron cloud with optical depth $\tau \ll 1$ and assume that the incident light is unpolarized and has some angular intensity pattern given by $I^{\text{in}}(\theta, \phi)$. The intensity of the light that is emitted in the z direction and polarized in the $\hat{\varepsilon}_i$ direction is then

$$I_i^{\text{out}} = (1 - \tau) I_i^{\text{in}}(\pi, 0) + \frac{\tau}{\sigma_T} \int d\Omega \; I^{\text{in}}(\theta, \phi) \frac{d\sigma_i}{d\Omega}(\hat{n}).$$

a) Recall that the Stokes parameter Q is defined as $Q \equiv I_x - I_y$ for light propagating in the z-direction. Show that

$$Q = \frac{-3\tau}{16\pi} \int d\Omega \; I^{\text{in}}(\theta, \phi) \sin^2(\theta) \cos(2\phi).$$

for light emitted in the z-direction from an electron cloud.

b) Recall that the Stokes parameter U may be defined as $U \equiv I_1 - I_2$ where $\hat{\varepsilon}_1 = (\hat{x} + \hat{y})/\sqrt{2}$ and $\hat{\varepsilon}_2 = (\hat{x} - \hat{y})/\sqrt{2}$. Show that

$$U = \frac{-3\tau}{16\pi} \int d\Omega \; I^{\text{in}}(\theta, \phi) \sin^2(\theta) \sin(2\phi).$$

for light emitted in the z-direction from an electron cloud.

c) The incident intensity field may be decomposed into spherical harmonics:

$$I^{\text{in}}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi).$$

Show that the polarization $P \equiv Q - iU$ is sourced by the quadrupole moment of the incident radiation field:

$$Q - iU = -\sqrt{\frac{3}{40\pi}} \tau a_{22}.$$

Thus we expect that an anisotropic radiation field will result in polarized emission.