23 Feb 2018 **Ay 127** E.S. Phinney & C.C. Steidel

Cosmology and Galaxy Formation

Problem Set 7

Due **in class**, Thursday 29 February 2018

Reading: See the on-line syllabus for lecture-by-lecture readings.

Collaboration policy: See the on-line collaboration policy.

Homework Problems:

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1. **"Pencil Beam" Redshift Surveys**

   Calculate the relation between the one-dimensional power spectrum $P_{\rm 1D}(k)$ (as would be measured in a pencil-beam survey, i.e., one which is small in the transverse direction but long in the line-of-sight direction) and the usual three-dimensional power spectrum $P(k)$.

2. **The Value of M_***

   Calculate (numerically) and plot the root-variance $\sigma(M)$ as a function of mass M for the CDM ($\Omega_{m,0} = 0.3$) power spectrum $P(k)$. Then, determine M_* (defined by $\sigma((M_*, t) = \delta_{\rm c}$ as a function of redshift from $z = 0$ to $z = 100$. Use the proper linear-theory growth factor (you can use the approximation given in class), and also the proper $\delta_{\rm c}$ (also using the approximation given in class).

3. **Characteristic Formation Redshifts of Dark Matter Halos**

   Suppose that the 1-point distribution function, normalized to unit variance, is $\mathcal{P}(\nu)$ (for Gaussian initial conditions, $\mathcal{P}(\nu) = (2\pi)^{-1/2}\exp\left(-\nu^2/2\right)$).

   (a) Show that the usual Press-Schechter equation for the number of gravitationally-bound halos with masses between M and $M + dM$ per comoving volume at redshift z generalizes to

   $$\frac{dn}{dM}dM = \frac{f\bar{\rho}}{M}\mathcal{P}\left[\nu(M,z)\right]\frac{\partial\nu(M,z)}{\partial M}dM \quad , \tag{1}$$

   where $\nu = \delta_{\rm c}(z)/\sigma(M)$, $\delta_{\rm c}(z) = 1.69/D(z)$ is the critical overdensity for gravitational collapse, $D(z)$ is the linear-theory growth factor, $\sigma(M)$ is the root variance of the mass distribution for scales M, and $f \equiv \int_0^\infty \mathcal{P}(\nu)d\nu$.

   (b) Now suppose that once halos form, their mass is fixed, and suppose further that they disappear only when they merge into larger halos. Show that with these assumptions, the distribution (normalized to unity) of formation redshifts $z_{\rm f}$ for halos of mass M observed at redshift z_0 is

   $$\frac{df}{dz_{\rm f}} = \mathcal{P}'[\nu(M,z_{\rm f})]\frac{\partial\nu(M,z_{\rm f})}{\partial z_{\rm f}}\left(\mathcal{P}[\nu(M,z_0)]\right)^{-1} \quad . \tag{2}$$

   (c) Evaluate this formation-redshift distribution for a Gaussian distribution of perturbations and describe it qualitatively. For hints, see MNRAS **321**, L7 (2001).