16 Feb 2018 **Ay 127** E.S. Phinney & C.C. Steidel

Cosmology and Galaxy Formation

Problem Set 6

Due **in class**, Thursday 22 February 2018

Reading: See the on-line syllabus for lecture-by-lecture readings. Note that this set is shorter since outlines for the writing project are also due this week.

Collaboration policy: See the on-line collaboration policy.

Homework Problems:

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1. **Weak Gravitational Lensing**

   Consider weak gravitational lensing of some background population of galaxies at a redshift $z \sim 1$ In the weak-lensing regime, the magnification is $\mu(\theta) = 1 + \epsilon(\vec{\theta})$ as a function of position $\vec{\theta}$ with $\epsilon(\vec{\theta}) \ll 1$. If $\epsilon > 0$, then you will be able to see fainter, and therefor more, sources. However, if $\epsilon > 0$, you are also magnifying the region behind the lens and thus seeing less volume per unit area on the sky. Suppose that a source population has a luminosity function $(dN/dL) \propto L^{-\alpha}$, where N is the volume number density. What is the condition on the value of α such that the sky surface density (number per unit solid angle) of background sources increases (decreases) for $\epsilon > 0$?

2. **Gravitationally Lensed Quasar**

   Suppose that you have discovered a gravitationally-lensed quasar with redshift $z = 2.7$, and observe that it produces two noticeable images of the quasar, separated by 2.2 arcsec on the plane of the sky, with one image observed to be $\simeq 1.5$ magnitudes fainter than the other. The images are superposed on a foreground galaxy with $z = 0.4$, strongly suggesting that the galaxy is the lens. Modeling the galaxy as a point mass, estimate the true position of the source relative to images A and B, and the mass of the lensing galaxy. By monitoring the system photometrically, you notice that one of the images has increased in brightness by 0.2 mag since the last measurement. Estimate how long it will be before the other image will show the same increase. You may assume that the time delay is roughly twice that expected from the difference in path lengths traveled for the two images.

3. **Dissecting the Intergalactic Medium with QSO Lensing**

   Much of what we know about the small-scale structure of gas in the intergalactic and circumgalactic medium comes from comparison of the details of the absorption lines from intervening gas recorded in the spectra of the separate images of gravitationally-lensed quasars. Taking the lensing geometry from problem 2, calculate and plot the difference in the physical separation of the two light paths as a function of redshift, starting at the redshift of the quasar $z_{\rm S} = 2.7$ and moving toward $z_{\rm L} = 0.0$ (hint: at $z = z_{\rm L}$, the paths are separated by 2.2 arcsec, so the distance separating the two light paths is the angular separation times $d_{\rm L}(z_{\rm L})$, where $d_{\rm L}$ is the angular diameter distance to redshift $z = 0.4$.)

4. **The Brightest Example of Anything is Probably Lensed**

   It is often the case in astrophysics that the first examples found of a new class of objects observed at high redshift ($z \gg 1$) are also the brightest. Experience has shown that sources with steep intrinsic luminosity distributions (like bright QSOs) are particularly affected at the highest apparent brightness by lensing magnification due to intervening masses. As a rough estimate, consider some source population at $z = 6$ and imagine that galaxies are point masses $M \simeq 10^{12}$ $M_\odot$ with a constant comoving space density of $\phi^* \simeq 4 \times 10^{-3}$ $\rm Mpc^{-3}$). Estimate the total strong lensing cross-section for a line of sight to $z \sim 6$ assuming it is dominated by the galaxies, and therefore the probability that a $z \sim 6$ object has been strongly lensed (i.e, $\theta_{\rm S} \le \theta_{\rm E}$). Using the expression for the magnification μ as a function of $\theta_{\rm S}/\theta_{\rm E}$ given in equation 6.211 on page 296 of the MvdBW text, what is the probability that a source has been magnified by more than a factor of 2? If the sources have an exponential (intrinsic) luminosity distribution at the bright end (e.g., as for a Schechter function), what is the probability that the (apparently) brightest known example has been lensed?