Problem Set 4

Due **in class**, Thursday 8 February 2018

Reading: See the on-line syllabus for lecture-by-lecture readings. Note that this set is shorter since outlines for the writing project are also due this week.

Collaboration policy: See the on-line collaboration policy.

Homework Problems:

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1. **Growth of Fluctuations**

   Consider linear growth of perturbations in a Universe with cold dark matter with density $\Omega_{\rm dm,0} = 0.25$ and baryon density $\Omega_{\rm b,0} = 0.05$. Consider only redshifts $z \gg 1$ so that the dynamical effect of Λ can be neglected. Write down the differential equations for linear evolution of $\delta_{\rm dm}(x,t) = \delta\rho_{\rm dm}(x,t)/\langle\rho_{\rm dm}\rangle$, the fractional perturbation to the CDM density, and for $\delta_{\rm b}(x,t) = \delta\rho_{\rm b}(x,t)/\langle\rho_{\rm b}\rangle$, the fractional perturbation to the baryon density. Now consider the evolution of a single Fourier mode of wavelength λ wavenumber $k = 2\pi/\lambda$, of the density field. Show that baryon perturbations are stabilized by pressure at small scales, and find an expression for the Jeans wavelength $\Lambda_{\rm J}$, the wavelength that separates stable and unstable modes. Evaluate $\Lambda_{\rm J}$ just before and just after recombination, and determine the corresponding Jeans mass.

2. **The CMB and the Baryon Acoustic Oscillation Scale**

   The physical size of the sound horizon at the epoch of recombination $d_{\rm h}(z_{\rm rec})$ is responsible for imprinting some of the most prominent features in the power spectrum of temperature fluctuations in the CMB, including the so-called "acoustic peaks". The angular scale of the first acoustic peak in the CMB power spectrum provides a powerful constraint on cosmological parameters because $d_{\rm h}$ can be treated as a "standard rod" whose length can be calculated, and the angular scale subtended by this length as seen by an observer at $z = 0$ is related to the angular diameter distance to $z_{\rm rec}$.

   (a) Write down an equation, preserving the dependence on h, and $\Omega_{\rm m,0}$, that describes the physical scale of the sound horizon assuming $z_{\rm rec} \sim 1100$. Write down another equation that relates the physical scale $d_{\rm h,rec}$ to the angle it should subtend on the sky as seen by an observer at $z = 0$. You may assume a Λ-CDM cosmology, and that we know the value of $\Omega_{\rm b,0}h^2$. Derive the joint constraints on $\Omega_{\rm m,0}$ (and therefore Ω_Λ) and h that would result from a measurement of the angular scale of the first acoustic peak.

   (b) What is the co-moving size of $d_{\rm h}(z_{\rm rec})$, in Mpc? Write down an equation that describes the expected angle subtended by this co-moving scale, as a function of $\Omega_{\rm m,0}$, h, and z.

   (c) It turns out that remnants of the co-moving baryon acoustic oscillation (BAO) scale can be detected as a small a $\sim 10\%$ "bump" of excess power in the large-scale correlation function/power spectrum of galaxies. Describe, using plots, words, or equations, how one might use galaxy surveys, with redshift "slices" judiciously chosen to sample several redshifts between $z = 0.1$ and $z \sim 3$, to measure the angular scale of the BAO "standard rod" and thereby constrain cosmological parameters.

   (d) Given the angular scale subtended by the BAO feature at $z \sim 1$, what is the minimum solid angle on the sky necessary for a redshift survey to be sensitive to the BAO scale?