Astronomy 127

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Cosmology and Galaxy formation

6 Apr 2007

TA office hours for this problem set

Adrienne Erickcek: Weds 5-6pm. Dan Grin: Thurs 5-7pm. If many students turn up, the TAs may move to 124 Bridge (”Interaction Room”).

Readings:

Week of Apr 9: Coles chapter 5 and Sections 8.1-8.6
and/or sections 3.1-3.2 of Dodelson (this unfortunately is a bit sketchy, so Coles is preferable).

Problem Set 2

Due in class *in class Fri, 13 Apr 2007*

Homework Problems:

1. One method astronomers use to try to determine the cosmological parameters is to attempt to identify ‘standard candles’ whose luminosities are known (or at least devoutly hoped) to be the same at all redshifts. Type Ia supernovae are the most recent objects of such religious veneration. The observed flux (in appropriately redshifted bands) from these ‘standard candles’ then gives luminosity distance $D_L(z)$ as a function of redshift z. Since the function $D_L(z)$ depends on cosmological parameters of the universe, observations of it can help constrain them. As described in the class handout (“remedial notes for physicists”), optical astronomers usually quote apparent magnitude instead of flux, and absolute magnitude (the apparent magnitude the object would have at the standard distance of 10 parsecs) instead of luminosity, so that instead of luminosity distance D_L they use the more convenient distance modulus $m - M = 5\log(D_L/10\text{pc})$. Suppose that observers measure $m - M$ of objects at $z = 0.1$, $z = 0.5$ and $z = 1$. Calculate D_L and $m - M$ at these three redshifts for
   a) an Einstein-de Sitter universe ($\Omega_m = 1$, $\Omega_\Lambda = 0$: standard 1990’s cosmology).
   b) an open universe with $\Omega_m = 0.26$ and $\Omega_\Lambda = 0$ (standard 1970-80’s cosmology).
   c) a flat universe with $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, and $w = -1$ (standard cosmology in 2007).
   d) a flat universe with $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, and $w = -0.9$ (¿standard cosmology in ?).
   In all cases use $H_0 = 100h\,\text{km s}^{-1}\text{Mpc}^{-1}$, with h as a free parameter. Then
   e) Determine the differences in the distance moduli between these four cosmologies at $z = 0.5$ and also at $z = 1$. Discuss how well these models could be distinguished by measuring the brightnesses of supernovae, with magnitude errors of (i) 0.15 mag (inferred actual dispersion of calibrated SN to calibrated SN) and (ii) 0.01 mag (dreams of variance in the mean of large samples of supernovae with perfectly understood astrophysics and the measurement errors of a future space mission).

2. The solar system is 4.6 billion years old. Suppose life on Mars evolved much more rapidly than on earth, so little green Martians (LGMs) were already observing supernovae at cosmological

distances 4.5 billion years ago. Assuming that the true universe has $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, $w = -1$, and $H_0 = 71\text{km s}^{-1}\text{Mpc}^{-1}$, repeat all the parts of problem 1 for the universe as observed by the LGMs 4.5 billion years ago. Also add

f) Explain qualititatively the origin of the difference in the difficulty we and the LGMs have in studying dark energy.

g) At the time the LGMs were observing them, what were the redshifts of the galaxies which we now say have $z = 0.5$ and $z = 1$?

3. In standard cosmology, (almost) all the electrons and ions in the Universe recombine to form neutral atoms by a redshift $z \sim 1000$. However, at some lower redshift z_{reion}, a few stars and black hole accretion disks form and emit radiation that ionizes all the remaining majority of hydrogen (but probably not helium) in the Universe. The resulting free electrons can Thomson scatter cosmic microwave background (CMB) photons.

   a) Calculate the optical depth $\tau(z_{\text{reion}}$ for Thomson scattering of CMB photons as a function of the redshift of reionization z_{reion} for a universe with $\Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, $\Omega_b = 0.044$, $h = 0.71$ and a helium mass fraction $Y = 0.23$.

   b) Derive an analytic approximation for $z_{\text{reion}} \gg \Omega_m^{-1}$ and verify that your exact expression agrees with this answer in this limit.

   c) At what z_{reion} does $\tau = 1$? At what z_{reion} does $\tau = 0.17$ (2003: WMAP year 1 result)? At what z_{reion} does $\tau = 0.09$ (2006: WMAP year 3 result)?

4. The radio galaxy 3C 179, at redshift $z = 0.846$, is observed to have knots of radio emission moving away from its center. The separation of these knots increases by 0.19 milliarcsec per year. If $H_0 = 100h\,\text{km s}^{-1}\text{Mpc}^{-1}$, $h = 0.71, \Omega_m = 0.26$, $\Omega_\Lambda = 0.74$, calculate the speed at which the knots are moving out. Be sure the speed you give is that which would be measured in the frame of cosmological observers near 3C 179. Compare the speed to well-known speed limits.

5.

   a) Show, in two ways, that the temperature of a non-relativistic gas of monatomic particles filling an expanding unverse decreases as $T_{nr}(t) \propto a(t)^{-2}$

      i) by considering the effect of expansion on the momentum of the individual particles.

      ii) by using the adiabatic equation of state for a monatomic gas.

   b) Repeat both parts of the previous problem for a gas of ultrarelativistic particles (kinetic energy $\gg mc^2$), showing that $T_r(t) \propto a(t)^{-1}$.

   c) What do you think would be the behavior of the temperature of a coupled mixture of a nonrelativistic gas and an ultrarelativistic gas, exchanging energy with each other in an expanding universe?