Astronomy 127

E. S. Phinney

Cosmology and Galaxy formation — 30 Mar 2007

TA office hours for this problem set

Dan Grin: Weds 5-6pm. Adrienne Erickcek: Thurs 5-7pm. If many students turn up, the TAs may move to 124 Bridge ("Interaction Room").

Readings:

Week of Mar 28: Coles chapter 1 (or Dodelson chapter 1)
Week of Apr 2: Coles chapters 2 and 4 (Dodelson chapter 2 unfortunately does not cover all the required material, so you need Coles for this part.)

Problem Set 1

Due in class *in class Fri, 6 Apr 2007*

Homework Problems:

1. **'Deriving' Hubble's law.**
   a) Local homogeneity requires that a generalized Hubble expansion law $\vec{v} = \vec{f}(\vec{r}, t)$ be the same for all observers. Show that this requires

   $$\vec{f}(\vec{r}_{CA}) = \vec{f}(\vec{r}_{BA} + \vec{r}_{CB}) = \vec{f}(\vec{r}_{BA}) + \vec{f}(\vec{r}_{CB}) \ ,$$

   where $\vec{r}_{CA}$ is the radius vector from galaxy A to galaxy C, and so on.
   b) Prove that a homogeneous universe must have a linear Hubble flow

   $$v_i = \sum_{i=1}^{3} H_{ij}(t) r_j \ ,$$

   where the subscripts i and j run over the cartesian components (x, y, z).
   c) Prove that the additional assumption of isotropy is required to get the observed Hubble law $v_i = H(t) r_i$. [Hint: a flow with vorticity is not isotropic].

2. **If stars were the only mass-energy, this would be a good approximation!** In general relativity, mass-energy curves space-time. So a space empty of matter and energy should not be curved. An FRW universe in which the density of matter and radiation vanishes ($\Omega_0 \to 0$) describes an empty space, and so this spacetime ("Milne (1932) spacetime") should be merely the Minkowski metric in disguise (in the jargon of GR, disguises are called "gauge transformations", or just "new coordinates"). Unmask this by finding that coordinate transformation.

   Specifying the problem more explicitly: the FRW metric of constant negative curvature is

   $$ds^2 = -c^2 dT^2 + a^2(T)(d\chi^2 + \sinh^2 \chi (d\theta^2 + \sin^2 \theta \, d\varphi^2))$$

a) Show that the dynamical equations for an empty $\Omega_0 = 0$ universe require the Hubble parameter $H^2 \propto a^{-2}$ and thus $a(T) \propto T$.

b) Find the coordinate transformation $r(T,\chi)$, $t(T,\chi)$ that converts this FRW metric to the familiar flat Minkowski space (in spherical coordinates):

$$ds^2 = -c^2 dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

3. **Implications of the CMB dipole anisotropy.**

a) Show that an observer moving with speed $v = \beta c$ through an isotropic black body radiation field (Planck spectrum) of temperature T_0 measures, at angle θ to her direction of motion, a Doppler-shifted spectrum with local temperature

$$\begin{aligned} T(\theta) &= T_0(1-\beta^2)^{1/2}/(1-\beta\cos\theta) \\ &\simeq T_0(1+\beta\cos\theta + (\beta^2/2)\cos 2\theta + \ldots)\,. \end{aligned}$$

b) As seen from the solar system barycenter, the cosmic microwave background (CMB) is 'observed' to have $T_0 = 2.725$K, and an $l = 1$ dipole anisotropy of $T_0\beta = 3.35$mK.

i) Explain why the dipole anisotropy is always quoted as if it were observed from the solar system barycenter instead as actually measured from earth (or earth-orbit)? How much would it vary over the year?

ii) What is v of the solar system barycenter, in km/s?

c) Because the sun is moving around the center of the galaxy at ~ 235km s^{-1} in approximately the opposite direction, the actual velocity of the Milky Way galaxy with respect to the CMB is ~ 600km s^{-1} [students who took Ay 124 might like to figure this out in detail: the solar system barycenter is moving relative to the CMB toward $l = 263.8°$, $b = 48.2°$ at the speed you found in part b-ii].

d) If the peculiar velocity of our Milky Way relative to the CMB is typical of other galaxies, how far away would a typical galaxy have to be before it could be used to determine the value of the Hubble constant (~ 71km s^{-1}Mpc^{-1}) to 10% accuracy? (You may assume that the galaxy distance could be measured exactly, which sadly is never the case in real astronomy!).

4. **Inhomogeneity in the present universe.** When computing, adopt the presently preferred (WMAP-3) $\Omega_m = 0.26$, $\Omega_b = 0.044$, $\Omega_k = 0$ $H_0 = 71$km s^{-1}Mpc^{-1}.

a) The mass within the sun's 8kpc orbital radius about the Milky Way is $\sim 8 \times 10^{10} M_\odot$.

i) By what factor does the average density within this sphere exceed the mean density of matter in the universe?

ii) But $\sim$ 70% of mass within the sun's 8kpc orbital radius is accounted for by the baryons of stars and gas. By what factor does the average baryon density within this sphere exceed the mean density of baryons in the universe? Can you think of a reason the baryons would concentrate more than the dark matter?

b) The mass of the Milky Way's dark halo is $2\times10^{12} M_\odot$, and that of the nearest comparably large galaxy, the Andromeda galaxy M31 is $1.2\times10^{12} M_\odot$. M31 is 780 kpc distant, falling towards the Milky Way at 116km s^{-1}. By what factor does the average density within a sphere of radius 780 kpc centered between the Milky Way and M31 exceed the mean density of matter in the universe?

5. **Age and Expansion history of our universe** For a flat universe composed of cold matter (aka 'dust', today making up a fraction Ω_m of the critical density) and cosmological constant (as appears to be an accurate representation of our Universe for redshifts $0 < z < 10^3$)

   a) Show that the age of the universe is

$$t_0 = \frac{2}{3H_0} \frac{\sinh^{-1} \sqrt{\frac{1-\Omega_m}{\Omega_m}}}{\sqrt{1-\Omega_m}} = \frac{2}{3H_0} \frac{\ln\left(\frac{1+\sqrt{1-\Omega_m}}{\sqrt{\Omega_m}}\right)}{\sqrt{1-\Omega_m}} .$$

   Evaluate this age, in years, for the cosmological parameters given in the previous problem.

   b) Show that the scale factor $a(t)$ as a function of time is given by

$$a(t) = a_0 \left(\sqrt{\frac{\Omega_m}{1-\Omega_m}} \sinh(\frac{3}{2}\sqrt{1-\Omega_m} H_0 t) \right)^{2/3} ,$$

   where a_0 is the present-day scale factor.