

Homework Set # 4

Due: Friday, 16 November 07

1. A source process $A(t)$ is passed through two token bucket regulators in tandem (series). The first has bucket size b_1 and token fill rate ρ_1 , while the second has bucket size b_2 and rate ρ_2 . Suppose $b_2 > b_1 > 0, \rho_1 > \rho_2 > 0$.
 - (a) Sketch the envelopes $E_1(t)$ and $E_2(t)$ imposed by the two individual regulators respectively, and find the overall envelope $E(t)$ imposed by the two regulators in tandem.
 - (b) The overall departure process $D(t)$ has envelope $E(t)$; show that it also has envelope $E_1(t)$.
 - (c) If process $D(t)$ declares that it has envelope $E_1(t)$, how much capacity should a constant fluid rate server allocate to guarantee that $D(t)$ experiences at most a delay of d passing through the server?
 - (d) If process $D(t)$ declared its actual envelope $E(t)$ instead, could the constant fluid rate server allocate less capacity for some given delay bound d ? If so, for what range of values of d is this the case?
2. **(extra credit)** A source process $A(t)$ with envelope $(b + \rho t)I_{\{t \geq 0\}}$ is passed through a system with maximum delay d (i.e., any fluid entering the system at time t leaves at or before time $t+d$). Show that the output process $D(t)$ has envelope $E(t) = (b_1 + \rho t)I_{\{t \geq 0\}}$ where $b_1 = b + \rho d$.