Characterization of stationary queue length with arbitrary buffering

- cumulative arrival process $A(t)$ with stationary increments (an increment of $A(t)$ is the difference $A(t+s) - A(t)$ corresponding to some interval $(t, t+s]$; stationary increments implies that for any set of increments of $A(t)$, the joint distribution is invariant to a shift of all the intervals by the same amount)
- let the link rate be C, let $D(t)$ denote the departure process, & let $X(t)$ denote the amount of data in the buffer
- suppose $X(0^-) = 0$
- $$X(t) = A(t) - D(t)$$
$$= A(t) - \inf_{0^- \le s \le t} (A(s) + C(t-s))$$
$$= \sup_{0^- \le s \le t} (A(t) - A(s) - C(t-s))$$
- We want to find the probability distribution of $X(t)$ for $t \to \infty$
- Suppose the arrival process extends over all time, & the buffer is empty at some time $(-t)^-$
  - in this case we denote the amt of data in the buffer by $X_{-t}(\cdot)$, & we have
$$X_{-t}(0) = \sup_{(-t)^- \le s \le 0} (A(0) - A(s) - C(-s))$$
  - by stationarity, $X(t)$ & $X_{-t}(0)$ have the same distribution
- As $t \uparrow_s$, for any sample path of the arrival process, the sup is over an $\uparrow$ g set, & thus $X_{-t}(0)$ is nondecreasing & converges, possibly to ∞
  - let $X_{-\infty}(0) = \sup_{\tau \ge 0} (A(0) - A(-\tau) - C\tau$ denote this limiting r.v.
  - the distribution of $X(t), t \to \infty$ converges to the distribution of $X_{-\infty}(0)$
- for the discrete time case, an analogous argument can be used to show that the distribution of the amount of data in the buffer, X_n, as $n \to \infty$, converges to the distribution of
$$\sup_{k \ge 0} (A_0 - A_{-k} - Ck)$$
where the arrival process extends over all time steps $k \in \mathbb{Z}$