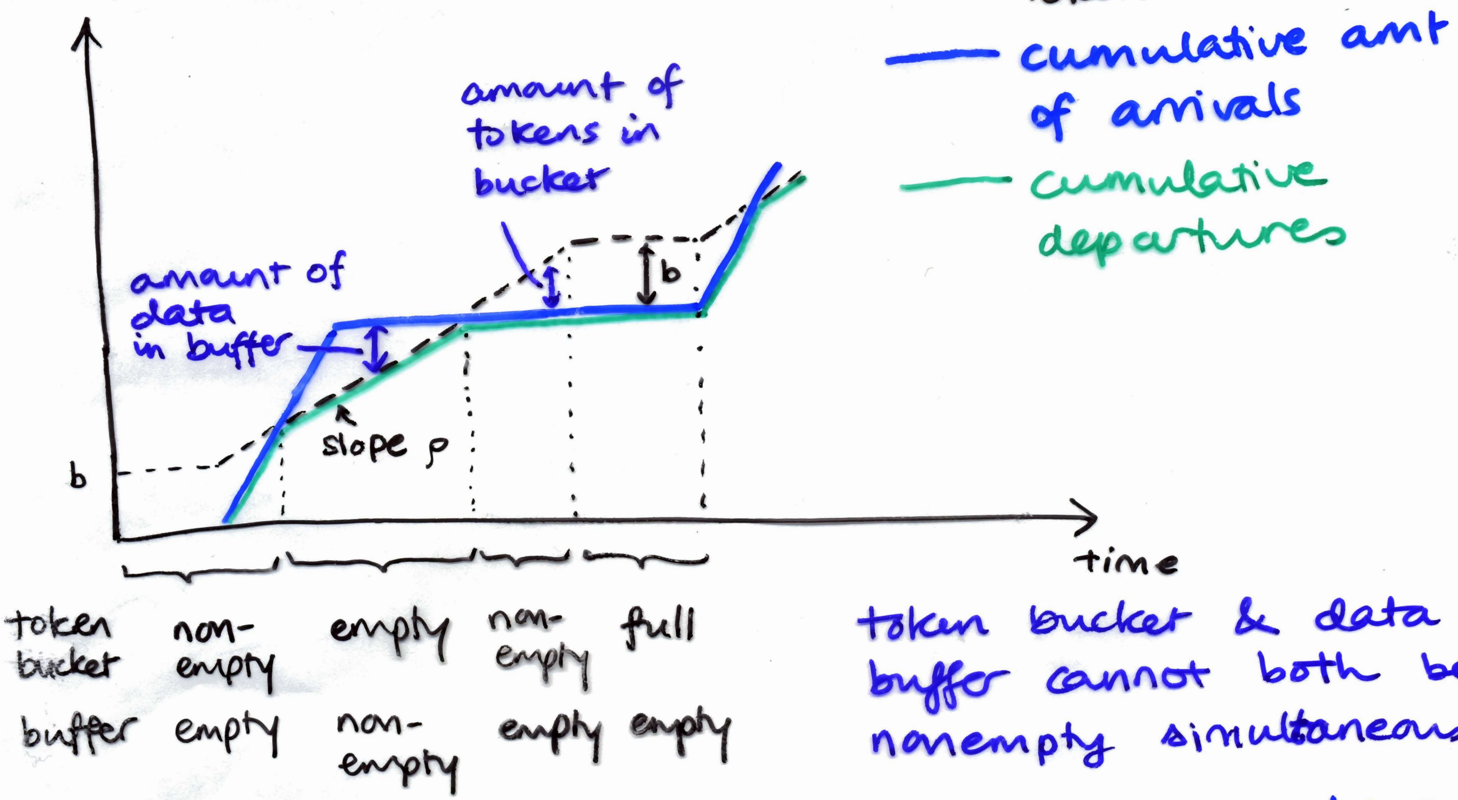
- Assume fluid arrivals & fluid tokens, causal arrival process



token bucket & data buffer cannot both be nonempty simultaneously

<u>Theorem</u>: The arrival process A & departure process D of an LB regulator are related by $D = A * E$, where $E(t) = \begin{cases} b + \rho t & t \geq 0 \\ 0 & t < 0 \end{cases}$

is the envelope of the regulator (i.e. the envelope $E(t)$ is the service curve of the LB regulator)

<u>Proof</u>:

- For any $0^- \leq \tau \leq t$,
$$D(t) = D(\tau) + D(t) - D(\tau) \leq A(\tau) + E(t - \tau)$$
$$\Rightarrow D(t) \leq \inf_{0^- \leq \tau \leq t} (A(\tau) + E(t - \tau)) = (A * E)(t) \text{ since } A, E \text{ causal}$$

- If the data buffer is empty at time t,

  $D(t) = A(t) \geq (A * E)(t)$ since $E \leq \delta$

- If the data buffer is nonempty at time t,

  let $v = \sup\{ s \leq t : \text{token bucket is full at time } s \}$

  $\rightarrow D(v) = A(v)$

  $$D(t) = D(v) + \underbrace{b + \rho(t-v)}$$

  avail.

  tokens available between v & t (all matched with departures)

  $$= A(v) + E(t-v)$$

  $$\geq \inf_{0 \leq \tau \leq t} \left(A(\tau) + E(t-\tau) \right)$$

  $$= (A * E)(t)$$

$\therefore$, $D(t) = (A * E)(t)$

$\square$

- <u>IntServ Tspec parameters</u>
  - token bucket rate ρ (average flow rate)
  - bucket depth b
  - peak rate p
  - max packet size M
  - minimum policed unit m (packets smaller than m are counted as being of size m for the purpose of policing a flow for conformance to the Tspec)

  $\rightarrow$ resulting envelope is $\min(M + pt,\ b + \rho t)$