- Service elements in tandem

\[
\begin{array}{c|c|c|c|c}
A(t) & S_1(t) & S_2(t) & \ldots & S_n(t) \\
\hline
D_1(t) & S_2(t) & \ldots & S_n(t) \\
\hline
D_n(t) & \text{lower service curve of } i\text{th element} \\
\hline
\end{array}
\]

- \( S_i(t) \): lower service curve of \( i\text{th element} \)

- \( \overline{S}_i(t) \): upper service curve of \( i\text{th element} \)

- \( D_n \geq D_{n-1} \ast S_n \)

- \( D_{n-1} \ast S_{n-1} \ast S_n \)

- \( \geq A \ast \overline{S}_1 \ast \overline{S}_2 \ast \ldots \ast \overline{S}_n \)

- Lower service curve for tandem

- \( D_n \leq D_{n-1} \ast \overline{S}_n \)

- \( \leq A \ast \overline{S}_1 \ast \overline{S}_2 \ast \ldots \ast \overline{S}_n \)

- Upper service curve for tandem

- Eg: lower service curve of a tandem of latency rate servers

- with \( S_i(t) = r_i (t - d_i)^+ \) being the lower service curve of the \( i\text{th server} \):

\[
(S_1 \ast S_2 \ast \ldots \ast S_n)(t) = (r_1 (t - d_i)^+) \ast \ldots \ast (r_n (t - d_i)^+) \]

\[
= (\min_{1 \leq i \leq n} r_i) (t - \sum_{i=1}^{n} d_i)^+ = r (t - d)^+
\]

where \( r = \min_{1 \leq i \leq n} r_i \), \( d = \sum_{i=1}^{n} d_i \)

- Envelopes

- Deterministic characterization of a flow's traffic characteristics

- Let \( A(t) \) be a nondecreasing \( k \) nonnegative function

- Let \( \{ \text{a cumulative} \} \) process \( A \) has the envelope \( E \) if

\[
\forall \tau \geq 0, A(t) - A(t-\tau) \leq E(t-\tau)
\]

- I.e. over any interval of time of length \( \tau \), the amount of data brought by process \( A \) is bounded by \( E(\tau) \)

- \( A \) has envelope \( E \) iff

\[
\int_{0}^{\tau} (A(t) + E(t-\tau)) dx = \sup_{\tau} (A(t) + E(t-\tau))
\]

- For causal \( A \) & \( E \),

\[
A \ast E \leq A \ast S
\]

\[
= A
\]

\[
\leq A \ast E
\]

\[
\Rightarrow A \ast E = A
\]

- Regulators

- A random source can be made to conform to a given envelope by passing it through a regulator

- A network element is a regulator with envelope \( E \) if for any arrival process \( A \), the departure process \( D \) has envelope \( E \) (\( D \leq D \ast E \))

\[
A(t) \xrightarrow{\text{regulator}} D(t)
\]
Buffered Leaky Bucket (LB) regulator / LB shaper

Token Bucket regulator

- Tokens are fed into a token bucket of size $b$ at rate $p$; tokens that overflow the bucket are discarded.

- Data is allowed to leave the regulator only if there are matching tokens in the token bucket; otherwise the data is nonconformant and is buffered until tokens become available.

- For any input process $A$, the output process satisfies $\forall 0 \leq t \leq T$

\[ D(t) - D(0) \leq b + p(T-t) \]

(in the interval $[t, T]$, the amount of data departing is upper bounded by the no. of tokens at $t$ plus the no. of tokens arriving in $[t, T]$)

$\rightarrow D$ has the envelope $E(t) = \begin{cases} b + pt & t \geq 0 \\ 0 & t < 0 \end{cases}$

- Alternative interpretation: a leaky bucket that is initially empty has size $b$ & leaks fluid at a rate $p$ whenever it is nonempty.

- Data is allowed to leave the regulator only if it can pour into the bucket an amount of fluid equal to the amount of data without overflow.

- A policer drops or is the priority class of nonconformant data.