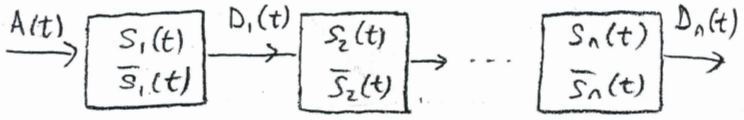
• service elements in tandem



$S_i(t)$: lower service curve of ith element

$\bar{S}_i(t)$: upper service curve of ith element

$$D_n \geq D_{n-1} * S_n$$
$$\geq D_{n-2} * S_{n-1} * S_n$$
$$\vdots$$
$$\geq A * \underbrace{S_1 * S_2 * \dots * S_n}$$

lower service curve for tandem

$$D_n \leq D_{n-1} * \bar{S}_n$$
$$\vdots$$
$$\leq A * \underbrace{\bar{S}_1 * \bar{S}_2 * \dots * \bar{S}_n}$$

upper service curve for tandem

- eg. lower service curve of a tandem of latency rate servers with $S_i(t) = r_i(t-d_i)^+$ being the lower service curve of the ith server:

$$(S_1 * S_2 * \dots * S_n)(t)$$
$$= (r_1 t^+ * \delta_{d_1}(t)) * \dots * (r_n t^+ * \delta_{d_n}(t))$$
$$= (\min_{1 \leq i \leq n} r_i) t^+ * \delta_{\sum_{i=1}^{n} d_i}(t)$$
$$= r(t-d)^+$$

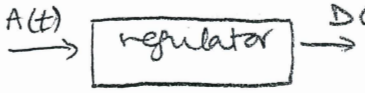
where $r = \min_{1 \leq i \leq n} r_i$, $d = \sum_{i=1}^{n} d_i$

• envelopes

- deterministic characterization of a flow's traffic characteristics
- let $E(t)$ be a nondecreasing & nonnegative function
- a (cumulative) process A has the envelope E if $\forall\, t$ & $\tau \in [0,t]$,
  $A(t) - A(\tau) \leq E(t-\tau)$
  i.e. over any interval of time of length u, the amount of data brought by process A is bounded by $E(u)$
- A has envelope E iff
  $A \leq A * E = \inf_{\tau} (A(\tau) + E(t-\tau))$
- for causal A & E,
  $A * E \leq A * \delta$
  $= A$
  $\leq A * E$
  $\Rightarrow A * E = A$

• regulators

- a random source can be made to conform to a given envelope by passing it through a regulator
- a network element is a regulator with envelope E if for any arrival process A, the departure process D has envelope E ($D \leq D * E$)



Buffered Leaky Bucket (LB) regulator / LB shaper / Token Bucket regulator

- tokens are fed into a token bucket of size b at rate ρ; tokens that overflow the bucket are discarded
- data is allowed to leave the regulator only if there are matching tokens in the token bucket; otherwise the data is nonconformant and is buffered until tokens become available
- for any input process A, the output process satisfies $\forall\ 0 \le \tau \le t$

$$D(t) - D(\tau) \le b + \rho(t-\tau)$$

(in the interval $[\tau, t]$, the amount of data departing is upper bounded by the no. of tokens at τ plus the no. of tokens arriving in $[\tau, t]$)

→ D has the envelope $E(t) = \begin{cases} b + \rho t & t \ge 0 \\ 0 & t < 0 \end{cases}$

- alternative interpretation: a leaky bucket that is initially empty has size b & leaks fluid at a rate ρ whenever it is nonempty
  - data is allowed to leave the regulator only if it can pour into the bucket an amount of fluid equal to the amount of data without overflow
- a policer drops or ↓s the priority class of nonconformant data