Single connection — deterministic analysis with finite bottleneck link buffer (Lakshman & Madhaw 97)

- Assume a bottleneck link of capacity C & finite buffer size B; fixed packet length L
- deterministic losses from buffer overflow → periodic cyclical evolution where each cycle contains 0–2 slow starts & 1 congestion avoidance phase
- Let Δ be the bandwidth × RT propagation delay product in units of packets (i.e. $\frac{RTPD \times C}{L}$)
- Assume Δ large enough that pipe does not become full during slow start

- Slow start

  - Consider rounds of length equal to the BRTT, $(\Delta+1)\frac{L}{C} \triangleq T$, indexed by $m \in \{0,1,2,\ldots\}$

time	pkt ACK'd	W	pkts released	buffer length
0		1	1	→ 1
T	1	2	2,3	→ 2
2T	2	3	4,5	→ 2 } −1 +2
$2T+\frac{L}{C}$	3	4	6,7	3
3T	4	5	8,9	→ 2
$3T+\frac{L}{C}$	5	6	10,11	3 } −1 +2
$3T+\frac{2L}{C}$	6	7	12,13	4
$3T+\frac{3L}{C}$	7	8	14,15	5

  - during the m^{th} round,
    - window grows to 2^m
    - buffer length reaches $2^{m-1}+1$
  - If link buffer $B \geq 2$, buffer overflow occurs in round

$$m = \lfloor \log_2(B-1) \rfloor + 2 \triangleq m_B$$

$$\left(\Rightarrow 2^{m-2}+1 \leq B < 2^{m-1}+1\right)$$

at time

$$m_B T + \underbrace{(B+1-2)}_{\text{buffer length when overflow occurs}} \frac{L}{C} = m_B T + (B-1)\frac{L}{C}$$

(buffer length at time $m_B T$)

when the window is

$$2^{m_B-1} + B \triangleq W_B$$

  - buffer loss occurs in slow-start iff $W_B \leq W_{th}$ (Congest^n Threshold) (which occurs if W_{th} is large or B is small → W_B small)
  - at the time of the loss, there are B packets in the buffer & $W_B - B - 1$ packets in transit ($W_B - 1$ outstanding packets preceding the lost packet)
  - the ACKs for these $W_B - 1$ packets will cause the window to grow to $W_B + W_B - 1 = 2W_B - 1$ before the loss is detected (or less if slow-start ends first)
  - new $W_{th} = \frac{2W_B - 1}{2} < W_B$

    → if 2 slow-starts occur successively, buffer loss does not occur the second time in slow-start

- since the window grows by 1 for each ACK during slow-start, starting from initial value 1, the total no. of packets transmitted successfully during a slow-start phase is approximately the window size at the end of the phase

• Congestion Avoidance

- let W_0 be the window size at the start of a congestion avoidance phase; assume $W_0 < \Delta+1$
- since the window ↑s by 1 each BRTT, the window reaches $\Delta+1$ & the RT pipe is filled after time

$$t_{ca\text{-}pipe\text{-}fill} = T(\Delta+1-W_0+1)$$

during which time the no. of packets transmitted is

$$W_0 + (W_0+1) + \ldots + (\Delta+1)$$

$$= \frac{(\Delta+1-W_0+1)(W_0+\Delta+1)}{2}$$

$$\triangleq n_{ca\text{-}pipe\text{-}fill}$$

- When the pipe is full, ACKs return at the bottleneck link rate C, & the RT time ↑s because of buffer queueing delay — the window now ↑s at a slower rate of $\frac{1}{W(t)}$ every packet transmission time ($\frac{L}{C}$)
- a continuous time approximation is convenient & reasonably accurate for this slower growth phase

$$\frac{dW}{dt} = \frac{C/L}{W(t)}$$

$$\Rightarrow \int W\,dW = \int \frac{C}{L}\,dt$$

$$\Rightarrow \frac{W^2}{2} = \frac{C}{L}t + k$$

- time taken for window to grow from $\Delta+1$ (when the pipe is filled) to $\Delta+B+1$ (when buffer overflow occurs) is

$$t_{ca\text{-}buffer\text{-}fill} = \frac{L}{C}\left(\frac{(\Delta+B+1)^2}{2} - \frac{(\Delta+1)^2}{2}\right)$$

during which time, since the pipe is full, the no. of packets transmitted is

$$n_{ca\text{-}buffer\text{-}fill} = \frac{(\Delta+B+1)^2 - (\Delta+1)^2}{2}$$

- at the time of the loss, there are $\Delta+B$ outstanding packets preceding the lost packet
→ the ACKs for these packets cause a total window ↑ of < 1 packet (since window size at the time of loss, $\Delta+B+1$, is larger than $\Delta+B$) & result in $\Delta+B$ new packets sent without further loss
→ the ACKs for these packets are duplicate ACKs