EE/Ge 157 b

Week 2

Polarimetric Synthetic Aperture Radar (2)
POLARIMETRIC SYNTHETIC APERTURE RADAR

COORDINATE SYSTEMS

- All matrices and vectors shown in this package are measured using the **backscatter alignment** coordinate system. This system is preferred when calculating radar-cross sections, and is used when measuring them:

![Coordinate System Diagram]

- Transmitting Antenna
- Receiving Antenna
- Scattering Object
POLARIMETRIC SYNTHETIC APERTURE RADAR
MATHMATICL CHARACTERIZATION OF SCATTERERS: SCATTERING MATRIX

- The radiated and scattered electric fields are related through the complex 2x2 scattering matrix:

\[ \mathbf{E}^{sc} = [\mathbf{S}] \mathbf{p}^{rad} \]

- The (complex) voltage measured at the antenna terminals is given by the scalar product of the receiving antenna polarization vector and the received wave electric field:

\[ V = \mathbf{p}^{rec} \cdot [\mathbf{S}] \mathbf{p}^{rad} \]

- The measured power is the magnitude of the (complex) voltage squared:

\[ P = VV^* = |\mathbf{p}^{rec} \cdot [\mathbf{S}] \mathbf{p}^{rad}|^2 \]

**NOTE:** Radar cross-section is proportional to power
POLARIMETRIC SYNTHETIC APERTURE RADAR
MATHEMATICAL CHARACTERIZATION OF SCATTERERS:
COVARIANCE MATRIX

• We can rewrite the expression for the voltage as follows:

\[ V = P^{\text{rec}} \cdot [S] P^{\text{rad}} \]
\[ = P_h^{\text{rec}} P_h^{\text{rad}} S_{hh} + P_h^{\text{rec}} P_v^{\text{rad}} S_{hv} + P_v^{\text{rec}} P_h^{\text{rad}} S_{vh} + P_v^{\text{rec}} P_v^{\text{rad}} S_{vv} \]
\[ = (P_h^{\text{rec}} P_h^{\text{rad}} \quad P_h^{\text{rec}} P_v^{\text{rad}} \quad P_v^{\text{rec}} P_h^{\text{rad}} \quad P_v^{\text{rec}} P_v^{\text{rad}}) \begin{pmatrix} S_{hh} \\ S_{hv} \\ S_{vh} \\ S_{vv} \end{pmatrix} \]
\[ = \tilde{A} \tilde{T} \]

• The first vector contains only antenna parameters, while the second contains only scattering matrix elements. Using this expression in the power expression, one finds

\[ P = V V^* = (\tilde{A} \tilde{T})(\tilde{T}^* \tilde{A}^*) = \tilde{A} \tilde{T} \tilde{T}^* \tilde{A}^* = A \cdot [C] A^*; \quad [C] = T \tilde{T}^* \]

• The matrix [C] is known as the covariance matrix of the scatterer.
The power expression can also be written in terms of the antenna Stokes vectors. First consider the following form of the power equation:

\[
P = \left( \mathbf{p}_{\text{rec}}^{\text{sc}} \cdot \mathbf{E}_{\text{sc}}^{\text{sc}} \right)^* \left( \mathbf{p}_{\text{rec}}^{\text{sc}} \cdot \mathbf{E}_{\text{sc}}^{\text{sc}} \right) \\
= \left( p_{h_{\text{rec}}}^{\text{rec}} E_{h_{\text{sc}}}^{\text{sc}} + p_{v_{\text{rec}}}^{\text{rec}} E_{v_{\text{sc}}}^{\text{sc}} \right) \left( p_{h_{\text{rec}}}^{\text{rec}} E_{h_{\text{sc}}}^{\text{sc}} + p_{v_{\text{rec}}}^{\text{rec}} E_{v_{\text{sc}}}^{\text{sc}} \right)^* \\
= \left( \begin{bmatrix}
    p_{h_{\text{rec}}}^{\text{rec}} \\
    p_{v_{\text{rec}}}^{\text{rec}} \\
    p_{h_{\text{rec}}}^{\text{rec}} \\
    p_{v_{\text{rec}}}^{\text{rec}}
\end{bmatrix} \cdot \begin{bmatrix}
    E_{h_{\text{sc}}}^{\text{sc}} & E_{h_{\text{sc}}}^{\text{sc}*} \\
    E_{v_{\text{sc}}}^{\text{sc}} & E_{v_{\text{sc}}}^{\text{sc}*}
\end{bmatrix} + \left( \begin{bmatrix}
    p_{v_{\text{rec}}}^{\text{rec}} \\
    p_{h_{\text{rec}}}^{\text{rec}} \\
    p_{v_{\text{rec}}}^{\text{rec}} \\
    p_{h_{\text{rec}}}^{\text{rec}}
\end{bmatrix} \cdot \begin{bmatrix}
    E_{v_{\text{sc}}}^{\text{sc}} & E_{v_{\text{sc}}}^{\text{sc}*} \\
    E_{h_{\text{sc}}}^{\text{sc}} & E_{h_{\text{sc}}}^{\text{sc}*}
\end{bmatrix} + \left( \begin{bmatrix}
    p_{h_{\text{rec}}}^{\text{rec}} \\
    p_{v_{\text{rec}}}^{\text{rec}} \\
    p_{h_{\text{rec}}}^{\text{rec}} \\
    p_{v_{\text{rec}}}^{\text{rec}}
\end{bmatrix} \cdot \begin{bmatrix}
    E_{h_{\text{sc}}}^{\text{sc}} & E_{h_{\text{sc}}}^{\text{sc}*} \\
    E_{v_{\text{sc}}}^{\text{sc}} & E_{v_{\text{sc}}}^{\text{sc}*}
\end{bmatrix} + \left( \begin{bmatrix}
    p_{v_{\text{rec}}}^{\text{rec}} \\
    p_{h_{\text{rec}}}^{\text{rec}} \\
    p_{v_{\text{rec}}}^{\text{rec}} \\
    p_{h_{\text{rec}}}^{\text{rec}}
\end{bmatrix} \cdot \begin{bmatrix}
    E_{v_{\text{sc}}}^{\text{sc}} & E_{v_{\text{sc}}}^{\text{sc}*} \\
    E_{h_{\text{sc}}}^{\text{sc}} & E_{h_{\text{sc}}}^{\text{sc}*}
\end{bmatrix}
\right)^*
\right)
= \mathbf{g}_{\text{rec}} \cdot \mathbf{X}
\]

The vector \( \mathbf{x} \) in the expression above is a function of the transmit antenna parameters as well as the scattering matrix elements.
Using the fact that $E^r = [S]p^{rad}$, it can be shown that $x$ can also be written as

$$x = [W]g^{rad}$$

where

$$[W] = \begin{pmatrix}
S_{hh}S_{hh}^* & S_{hv}S_{hv}^* & S_{hh}S_{hv}^* & S_{hv}S_{hh}^* \\
S_{vh}S_{vh}^* & S_{vv}S_{vv}^* & S_{vh}S_{vv}^* & S_{vv}S_{vh}^* \\
S_{hh}S_{vh}^* & S_{hv}S_{vv}^* & S_{hh}S_{vv}^* & S_{vv}S_{hh}^* \\
S_{vh}S_{hh}^* & S_{vv}S_{hv}^* & S_{vh}S_{hv}^* & S_{hv}S_{vv}^*
\end{pmatrix}$$

This means that the measured power can also be expressed as:

$$P = g^{rec} \cdot [W]g^{red}$$
POLARIMETRIC SYNTHETIC APERTURE RADAR
MATHEMATICAL CHARACTERIZATION OF SCATTERERS:
STOKES SCATTERING OPERATOR

• From the earlier definition of the Stokes vector, we note that the Stokes vector can be written as:

\[
S = \begin{pmatrix}
    p_h^* + p_v^* \\
    p_h^* - p_v^* \\
    p_h^* + p_v^* \\
    -i(p_h^* - p_v^*) \\
\end{pmatrix} = \begin{pmatrix}
    1 & 1 & 0 & 0 \\
    1 & -1 & 0 & 0 \\
    0 & 0 & 1 & 1 \\
    0 & 0 & -i & i \\
\end{pmatrix} \begin{pmatrix}
    p_h^* \\
    p_v^* \\
    p_h^* \\
    p_v^* \\
\end{pmatrix} = [R]g \Rightarrow g = [R]^{-1}S
\]

• This means that we can express the measured power as:

\[
P = \tilde{S}^{\text{rec}} \cdot [R]^{-1}[W][R]^{-1} S^{\text{rad}} = \tilde{S}^{\text{rec}} \cdot [M]S^{\text{rad}}
\]

• The matrix \([M]\) is known as the Stokes scattering operator. It is also called Stokes matrix.
Once the scattering matrix, covariance matrix, or the Stokes matrix is known, one can *synthesize* the received power for any transmit and receive antenna polarizations using the polarization synthesis equations:

**Scattering matrix:**
\[ P = |\mathbf{p}_{\text{rec}} \cdot [\mathbf{S}][\mathbf{p}_{\text{rad}}]|^2 \]

**Covariance Matrix:**
\[ P = \mathbf{A} \cdot [\mathbf{C}]\mathbf{A}^* \]

**Stokes scattering operator:**
\[ P = \mathbf{S}_{\text{rec}} \cdot [\mathbf{M}]\mathbf{S}_{\text{rad}} \]

- Keep in mind that all matrices in the polarization synthesis equations must be expressed in the *backscatter alignment* coordinate system.
POLARIMETRIC SYNTHETIC APERTURE RADAR

Signatures
To fully characterize the scatterer, one must measure the full scattering matrix:

\[
\begin{pmatrix}
E_h \\
E_v
\end{pmatrix}^{rec} = \begin{pmatrix}
S_{hh} & S_{hv} \\ S_{vh} & S_{vv}
\end{pmatrix} \begin{pmatrix}
E_h \\
E_v
\end{pmatrix}^{tr}
\]

Setting one of the elements of the transmit vector equal to zero allows one to measure two components of the scattering matrix at a time:

\[
\begin{pmatrix}
S_{hh} \\
S_{vh}
\end{pmatrix} = \begin{pmatrix}
S_{hh} & S_{hv} \\ S_{vh} & S_{vv}
\end{pmatrix} \begin{pmatrix}
1 \\
0
\end{pmatrix}^{inc};
\begin{pmatrix}
S_{hv} \\
S_{vv}
\end{pmatrix} = \begin{pmatrix}
S_{hh} & S_{hv} \\ S_{vh} & S_{vv}
\end{pmatrix} \begin{pmatrix}
0 \\
1
\end{pmatrix}^{inc}
\]

This technique is commonly used to implement airborne and spaceborne SAR polarimeters, such as AIRSAR and SIR-C.
POLARIMETRIC SYNTHETIC APERTURE RADAR
POLARIMETER IMPLEMENTATION

**BLOCK DIAGRAM**

- Transmitter
- Receiver
- Horizontal
- Vertical

**TIMING**

Transmission:
- Horizontal
- Vertical

Reception:
- Horizontal: HH, HV, HH, HV, HH
- Vertical: VH, VV, VH, VV, VH
POLARIMETRIC SYNTHETIC APERTURE RADAR
POLARIZATION SIGNATURE OF DISTRIBUTED SCATTERERS

- The previous examples showed polarization signatures of single scatterers. It can be shown that each of these examples exhibit up to two nulls in the copolarized signature.
- When forming multi-looked images, power is averaged to reduce speckle noise at the expense of spatial resolution.
- Mathematically, the multi-looking operation can be written as:

\[
P_m = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} S_{rec} \cdot [M_i] S_{rad}^i = S_{rec} \cdot \left( \sum_{i=1}^{N} [M_i] \right) S_{rad}
\]

- This means that the multi-looked polarization signature can be considered as the sum of the individual polarization signatures.
- The only way the multi-looked polarization signature can exhibit a null is if all the individual signatures that were added exhibited the same null.
- In general, the multi-looked signature will exhibit minima rather than nulls, and will appear to “sit on a pedestal.” The height of the pedestal is a measure of how different the individual polarization signatures are.
POLARIMETRIC SYNTHETIC APERTURE RADAR

Effect of Phase Error on Response of Trihedral Corner Reflector

No Error

45 Degrees

90 Degrees

180 Degrees

135 Degrees
POLARIMETRIC SYNTHETIC APERTURE RADAR

OBSERVED POLARIZATION SIGNATURES:
SAN FRANCISCO

P-BAND

L-BAND

C-BAND

OCEAN

URBAN

PARK
At each frequency the ocean area scatters in a manner consistent with models of slightly rough surface scattering, the urban area like a dihedral corner reflector, while the park and natural terrain regions scatter much more diffusely, that is, the signatures possess large pedestals. This indicates the dominant scattering mechanisms responsible for the backscatter for each of the targets. The ocean scatter is predominantly single bounce, slightly rough surface scattering. The urban regions are characterized by two-bounce geometry as the incident waves are twice forward reflected from the face of a building to the ground and back to the radar, or vice versa. The apparent diffuse nature of the backscatter from the park and natural terrain indicates that in vegetated areas there exists considerable variation from pixel to pixel of the observed scattering properties, leading to the high pedestal. This variation may be due to multiple scatter or to a distinct variation in dominant scattering mechanism between 10m resolution elements.
POLARIMETRIC SYNTHETIC APERTURE RADAR
OBSERVED POLARIZATION SIGNATURES:
L-BAND POLARIZATION SIGNATURES OF THE OCEAN

20° INCIDENCE ANGLE

MODEL RESULTS

50° INCIDENCE ANGLE

MODEL RESULTS

OBSERVATION

OBSERVATION
POLARIMETRIC SYNTHETIC APERTURE RADAR

OBSERVED POLARIZATION SIGNATURES:

GEOLOGY

P-BAND

L-BAND

C-BAND

PLAYA

ALLUVIAL SURFACE

LAVA

Jakob van Zyl
The previous viewgraph shows polarization signatures extracted from an AIRSAR image of the Pisgah lava flow in California. Signatures correspond to the flow itself (a very rough surface), an alluvial fan (medium roughness), and from the playa next to the flow (a very smooth surface). Note that as the roughness of the surface increases, so does the observed pedestal height. This is quite consistent with the predictions of the slightly rough surface models, even though the surface r.m.s. heights exceed the strict range of validity of the model. The exception is the playa case, where the pedestal at P-band is higher than that at L-band. Possible explanations for this behavior include subsurface scattering due to increased penetration at P-band, or signal-to-noise limitations for the very smooth surface.
POLARIMETRIC SYNTHETIC APERTURE RADAR
OBSERVED POLARIZATION SIGNATURES:
SEA ICE

- P-BAND
- L-BAND
- C-BAND

FIRST YEAR | MULTIYEAR
POLARIMETRIC SYNTHETIC APERTURE RADAR
OBSERVED POLARIZATION SIGNATURES:
SEA ICE

Note the pedestal height as a function of frequency for the multi-year ice - the P-band pedestal is quite small, while the C-band pedestal is the greatest of the three. For the first year ice, exactly the opposite behavior is seen. The P-band signature shows the highest and the C-band signature the lowest pedestal. The multi-year ice behavior may be explained if we consider the ice to be formed of two layers, where the upper layer consists of randomly oriented oblong inclusions about the size of a C-band wavelength, several centimeters. The lower layer forms a solid, but slightly rough surface. In this situation the C-band signal would interact strongly with the diffuse scattering upper layer, giving rise to the high pedestal. The longer wavelength L- and P-band signals would pass through the upper and be scattered by the lower layer, which is smooth enough to exhibit fairly polarized backscatter. On the other hand, similar characteristics are also observed for simple rough surface scattering also, as previously explained.

At present we have no model to explain the first year ice behavior.
POLARIMETRIC SYNTHETIC APERTURE RADAR

OBSERVED POLARIZATION SIGNATURES:

VEGETATION

P-BAND

L-BAND

C-BAND

CLEAR-CUT

FOREST
POLARIMETRIC SYNTHETIC APERTURE RADAR
OBSERVED POLARIZATION SIGNATURES:
VEGETATION

The previous viewgraph polarization signatures for the heavily forested area and the clear-cut area of an area near Mt. Shasta in California. We note that all the signatures appear to be composed of a variable portion sitting on top of a large pedestal. The pedestal height varies with the type of scatterer, the heavy forest exhibiting a larger pedestal and the clear-cut a smaller pedestal. As discussed before, the pedestal is due to spatial variations in the observed scattering properties. From this we conclude that the returns from vegetated areas vary much more from pixel to pixels than the other terrain types discussed earlier. We also note that the pedestals increase with increasing wavelength due to increased penetration, plus the fact that the ratio of the size of the vegetation components relative to the wavelength is smaller for the longer wavelength.
Pixels dominated by odd numbers of reflections are typically characterized by

\[
|S_{hh}| \geq |S_{hv}|; \quad |S_{vv}| \geq |S_{hv}|; \quad \Re \{S_{hh}^* S_{vv}\} \geq |S_{hv}|^2; \quad \text{Phase}(S_{hh}^* S_{vv}) \approx 0
\]
THEORETICAL CHARACTERISTICS: EVEN NUMBERS OF REFLECTIONS

Pixels dominated by odd numbers of reflections are typically characterized by

\[ \langle |S_{hh}| \rangle \geq \langle |S_{hv}| \rangle; \quad \langle |S_{vv}| \rangle \geq \langle |S_{hv}| \rangle; \quad \text{Re}(S_{hh}^* S_{vv}) \geq \langle |S_{hv}|^2 \rangle; \quad \text{Phase}(S_{hh}^* S_{vv}) \approx \pi \]
Pixels dominated by diffuse scattering are typically characterized by

\[
\langle |S_{hh}| \rangle \geq \langle |S_{hv}| \rangle; \quad \langle |S_{vv}| \rangle \geq \langle |S_{hv}| \rangle; \quad \langle \Re(S_{hh}^* S_{vv}) \rangle < \langle |S_{hv}|^2 \rangle; \quad \text{Phase}(S_{hh}^* S_{vv}) \approx \text{uniform}[0, 2\pi]
\]
The definition of “moderate” vegetation is a function of the frequency used when imaging the scene and can be shown to be related to the randomness of the orientation and the thickness of the scattering cylinders relative to the radar wavelength.

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POLARIMETRIC SYNTHETIC APERTURE RADAR

Black Forest, Germany
POLARIMETRIC SYNTHETIC APERTURE RADAR

Black Forest, Germany
POLARIMETRIC SYNTHETIC APERTURE RADAR

Black Forest, Germany

C-Band

L-Band

P-Band
POLARIMETRIC SYNTHETIC APERTURE RADAR

Black Forest, Germany

C-Band

L-Band

P-Band
POLARIMETRIC SYNTHETIC APERTURE RADAR

Black Forest, Germany

C-Band  L-Band  P-Band
POLARIMETRIC SYNTHETIC APERTURE RADAR

Landes Forest, France
The average Stokes matrix of azimuthally symmetric terrain can be approximated by

$$\mathbf{[M]} = \frac{1}{4} \begin{pmatrix}
A + B_0 & B & 0 & 0 \\
B & A - B_0 & 0 & 0 \\
0 & 0 & C + B_0 & D \\
0 & 0 & D & B_0 - C
\end{pmatrix}$$

where

$$A = \langle |S_{hh}|^2 + |S_{vv}|^2 \rangle$$
$$B = \langle |S_{hh}|^2 - |S_{vv}|^2 \rangle$$
$$B_0 = 2\langle |S_{hv}|^2 \rangle$$
$$C = 2\langle \text{Re}(S_{hh}^* S_{vv}) \rangle$$
$$D = 2\langle \text{Im}(S_{hh}^* S_{vv}) \rangle$$

In these expressions, the angular brackets denote ensemble averaging.
POLARIMETRIC SYNTHETIC APERTURE RADAR

Landes Forest, France
POLARIMETRIC SYNTHETIC APERTURE RADAR

Landes Forest, France

C-Band  L-Band  P-Band
CLOUDE’S DECOMPOSITION THEOREM

- Cloude showed that a general covariance matrix $[T]$ can be decomposed as follows:

$$[T] = \lambda_1 k_1 \cdot k_1^\dagger + \lambda_2 k_2 \cdot k_2^\dagger + \lambda_3 k_3 \cdot k_3^\dagger + \lambda_4 k_4 \cdot k_4^\dagger$$

- Here, $\lambda_i, i = 1, 2, 3, 4$ are the eigenvalues of the covariance matrix, $k_i, i = 1, 2, 3, 4$ are its eigenvectors, and $k_i^\dagger$ means the adjoint (complex conjugate transposed) of $k_i$.

- In the monostatic (backscatter) case, the covariance matrix has one zero eigenvalue, and the decomposition results in at most three nonzero covariance matrices.
Also useful in our discussions later is Cloude's definition of target entropy,

\[ H_T = \sum_{i=1}^{4} -P_i \log_4 (P_i) \]

where

\[ P_i = \frac{\lambda_i}{\sum_{j=1}^{4} \lambda_j} \]

As pointed out by Cloude, the target entropy is a measure of target disorder, with \( H_T = 1 \) for random targets and \( H_T = 0 \) for simple (single) targets.
Borgeaud et al. showed, using a random medium model, that the average covariance matrix for azimuthally symmetrical terrain in the monostatic case has the general form

\[
[T] = C \begin{pmatrix}
1 & 0 & \rho \\
0 & \eta & 0 \\
\rho^* & 0 & \zeta
\end{pmatrix}
\]

where

- \( C = \langle S_{hh}S_{hh}^* \rangle \)
- \( \rho = \frac{\langle S_{hh}S_{vv}^* \rangle}{\langle S_{hh}S_{hh} \rangle} \)
- \( \eta = \frac{2\langle S_{hv}S_{hv}^* \rangle}{\langle S_{hh}S_{hh}^* \rangle} \)
- \( \zeta = \frac{\langle S_{vv}S_{vv}^* \rangle}{\langle S_{hh}S_{hh}^* \rangle} \)

The superscript * means complex conjugate, and all quantities are ensemble averages. The parameters \( C, \eta, \zeta \) and \( \rho \) all depend on the size, shape and electrical properties of the scatterers, as well as their statistical angular distribution.
The eigenvalues of $[T]$ are

$$
\lambda_1 = \frac{C}{2} \left( \zeta + 1 + \sqrt{(\zeta - 1)^2 + 4|\rho|^2} \right)
$$

$$
\lambda_2 = \frac{C}{2} \left( \zeta + 1 - \sqrt{(\zeta - 1)^2 + 4|\rho|^2} \right)
$$

$$
\lambda_3 = C\eta
$$

Note that the three eigenvalues are always real numbers greater than or equal to zero.
POLARIMETRIC SYNTHETIC APERTURE RADAR
CLOUDE’S DECOMPOSITION THEOREM
AZIMUTHALLY SYMMETRIC NATURAL TERRAIN

The corresponding three eigenvectors are

\[
k_1 = \sqrt{\frac{[\zeta - 1 + \sqrt{\Delta}]^2}{[\zeta - 1 + \sqrt{\Delta}]^2 + 4\rho^2}} \begin{pmatrix} 2\rho \\ \frac{2\rho}{(\zeta - 1 + \sqrt{\Delta})} \\ 0 \\ 1 \end{pmatrix}
\]

\[
k_2 = \sqrt{\frac{[\zeta - 1 - \sqrt{\Delta}]^2}{[\zeta - 1 - \sqrt{\Delta}]^2 + 4\rho^2}} \begin{pmatrix} 2\rho \\ \frac{2\rho}{(\zeta - 1 - \sqrt{\Delta})} \\ 0 \\ 1 \end{pmatrix}
\]

\[
k_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}
\]
POLARIMETRIC SYNTHETIC APERTURE RADAR
CLOUDE’S DECOMPOSITION THEOREM
AZIMUTHALLY SYMMETRIC NATURAL TERRAIN

- On the previous page we used the shorthand notation
  \[ \Delta = (\zeta - 1)^2 + 4|\rho|^2 \]

- We note that \( \Delta \) is always positive. Also note that we can write
  \[ \frac{k_{11}}{k_{21}} = \frac{(\zeta - 1 - \sqrt{\Delta})^2}{4|\rho|^2} K \]

- where
  \[ K = \sqrt{\left[\frac{\zeta - 1 + \sqrt{\Delta}}{\zeta - 1 - \sqrt{\Delta}}\right]^2 \left(\frac{\zeta - 1 + \sqrt{\Delta}}{\zeta - 1 - \sqrt{\Delta}} + 4|\rho|^2\right)} \]

- Since \( K \) is always positive, it follows that the ratio of \( k_{11} \) to \( k_{21} \) is always negative. This means that the first two eigenvectors represent scattering matrices that can be interpreted in terms of odd and even numbers of reflections.
POLARIMETRIC SYNTHETIC APERTURE RADAR
Example of Eigenvalue Decomposition
San Francisco, California
In general, the scattering matrix of a single dielectric cylinder oriented horizontally can be written as

$$ [S] = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} $$

where $a$ and $b$ are complex numbers whose magnitudes and phases are functions of cylinder dielectric constant, diameter and length.

Assuming a uniform distribution in angles about the line of sight, one can easily show that the resulting average covariance matrix for the monostatic case has the following parameters

$$ C = \frac{1}{8} \left[ 3|a|^2 + 3|b|^2 + 2 \Re(a^*b) \right] $$

$$ \rho = \frac{|a|^2 + |b|^2 + 6\Re(a^*b)}{3|a|^2 + 3|b|^2 + 2\Re(a^*b)} $$

$$ \eta = \frac{2|a - b|^2}{3|a|^2 + 3|b|^2 + 2\Re(a^*b)} $$

$$ \zeta = 1 $$
POLARIMETRIC SYNTHETIC APERTURE RADAR
CLOUDE'S DECOMPOSITION THEOREM
RANDOMLY ORIENTED DIELECTRIC CYLINDERS

• The eigenvalues are:

\[ \lambda_1 = C(1 + |\rho|) \quad \lambda_2 = C(1 - |\rho|) \quad \lambda_3 = C\eta \]

• The corresponding three eigenvectors are

\[ k_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho/|\rho| \\ 0 \\ 1 \end{pmatrix} \quad k_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} -\rho/|\rho| \\ 0 \\ 1 \end{pmatrix} \quad k_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \]
In the thin cylinder limit, \( b \to 0 \), and we find that

\[
\rho_{\text{thin}} = \frac{1}{3}
\]

\[
\eta_{\text{thin}} = \frac{2}{3}
\]

\[
[T] = C \left[ \frac{2}{3} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{pmatrix} + \frac{2}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right]
\]

In this case, equal amounts of scattering is contributed by the matrix that resembles scattering by a sphere and by the cross-polarized return, although a significant fraction of the total energy is also contained in the second matrix, which resembles a metal dihedral corner reflector.

The entropy in this case is 0.95 indicating a high degree of target disorder or randomness.

Note that the unsupervised classification scheme would classify this as *diffuse scattering*. 
In the thick cylinder limit, $b \rightarrow a$ and we find that
\[ p_{\text{thick}} = 1 \]
\[ n_{\text{thick}} = 0 \]

In this case, only one eigenvalue is non-zero, and the average covariance matrix is identical to that of a sphere.

The entropy is 0, indicating no target randomness, even though we have calculated the average covariance matrix for randomly oriented thick cylinders!

The explanation for this result lies in the fact that when the cylinders are thick, the single cylinder scattering matrix becomes the identity matrix, which is insensitive to rotations.

Note that the unsupervised classification scheme would classify this as *odd numbers of reflections.*
POLARIMETRIC SYNTHETIC APERTURE RADAR
CLOUDE'S DECOMPOSITION THEOREM
RADAR THIN VEGETATION INDEX

• Using the result for a cloud of randomly oriented thin cylinders, we note that

\[
\frac{\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} = \frac{1}{4}
\]

• We now define a radar thin vegetation index (RVI) as

\[
RVI = \frac{4\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} = \frac{8\langle S_{hv}S_{hv}^* \rangle}{\langle S_{hv}S_{hv}^* \rangle + \langle S_{vv}S_{vv}^* \rangle + 2\langle S_{hv}S_{hv}^* \rangle} \approx \frac{4\mu_i}{1 + \mu_i} \quad \mu_i = \frac{\langle S_{hv}S_{hv}^* \rangle}{\langle S_{vv}S_{vv}^* \rangle}
\]

• We expect RVI to vary between 0 and 1.
POLARIMETRIC SYNTHETIC APERTURE RADAR
CLOUDE'S DECOMPOSITION THEOREM
RADAR THIN VEGETATION INDEX

\[
[S] = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}
\]

Ratio of \(b\) to \(a\)
**POLARIMETRIC SYNTHETIC APERTURE RADAR**

**CLOUDE'S DECOMPOSITION THEOREM: EXAMPLE**

**VEGETATED CLEARCUT AREA**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>P-Band</th>
<th>L-Band</th>
<th>C-Band</th>
</tr>
</thead>
<tbody>
<tr>
<td>η</td>
<td>0.5621</td>
<td>0.5308</td>
<td>0.4083</td>
</tr>
<tr>
<td>ζ</td>
<td>0.5642</td>
<td>0.7580</td>
<td>0.7159</td>
</tr>
<tr>
<td>ρ</td>
<td>0.0928+i0.0582</td>
<td>0.2324+i0.1057</td>
<td>0.3558+i0.0440</td>
</tr>
<tr>
<td>λ₁</td>
<td>1.0260</td>
<td>1.1615</td>
<td>1.2437</td>
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<tr>
<td>λ₂</td>
<td>0.5382</td>
<td>0.5964</td>
<td>0.4722</td>
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<tr>
<td>λ₃</td>
<td>0.5261</td>
<td>0.5308</td>
<td>0.4083</td>
</tr>
<tr>
<td>Entropy</td>
<td>0.95</td>
<td>0.94</td>
<td>0.88</td>
</tr>
<tr>
<td>RVI</td>
<td>1.01</td>
<td>0.93</td>
<td>0.77</td>
</tr>
</tbody>
</table>
The clearcut area is covered with short shrub-like vegetation. We note that at all three frequencies the scattering is dominated by an odd number of reflections, i.e. the first eigenvalue is the dominant one. At P-band the even number of reflections (second eigenvalue) and the cross-polarized returns (third eigenvalue) are almost the same strength, and about half that of the odd numbers of reflections. This is very similar to the thin randomly oriented cylinder case discussed earlier. As the frequency increases, the even number of reflections and the cross-polarized returns become more different, and also become a smaller fraction of the total scattering. This is consistent with the randomly oriented cylinder case where the radius of the cylinder increases. The same conclusion is reached when considering the entropy. The highest value (most randomness) is observed at P-band, and the randomness decreases with increasing frequency. As pointed out before, as the cylinder radius increases, the entropy decreases. Therefore, from the decomposition results we conclude that the vegetation in the clearcut is randomly oriented, and that most of the scattering comes from vegetation that have branches that are thin compared to the three radar wavelengths.
POLARIMETRIC SYNTHETIC APERTURE RADAR

Black Forest, Germany

C-Band  L-Band  P-Band
POLARIMETRIC SYNTHETIC APERTURE RADAR

CLOUDE’S DECOMPOSITION THEOREM

• Advantages
  – Rigorous mathematical technique
  – Provides quantitative information about scattering mechanisms

• Disadvantages
  – Not physically based
  – Interpretation of results not unique