EE/Ae 157a

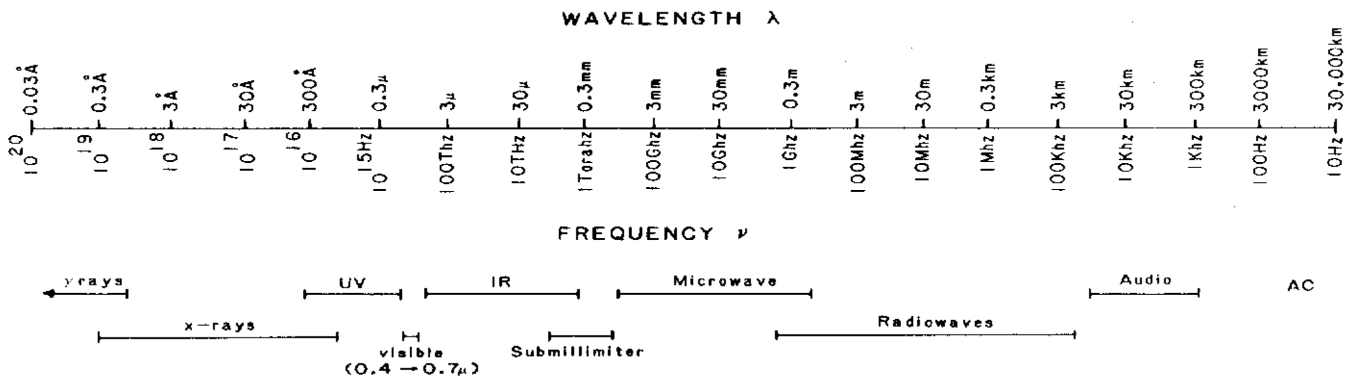
Introduction to the Physics and Techniques of Remote Sensing

Week 2: Nature and Properties of Electromagnetic Waves

TOPICS TO BE COVERED

- Fundamental Properties of Electromagnetic Waves
  - Electromagnetic Spectrum, Maxwell's Equations, Wave Equation, Quantum Properties of EM Radiation, Polarization, Coherency, Group and Phase Velocity, Doppler Effect
- Nomenclature and Definition of Radiation Quantities
  - Radiation Quantities, Spectral Quantities, Luminous Quantities
- Generation of Electromagnetic Radiation
- Detection of Electromagnetic Radiation
- Overview of Interaction of EM Waves with Matter
- Interaction Mechanisms Throughout the Electromagnetic Spectrum

ELECTROMAGNETIC SPECTRUM



MAXWELL'S EQUATIONS

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H}$$

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

WAVE EQUATION

From Maxwell's Equations, we find:

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \mu_r \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$$

$$= -\mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla \times \nabla \times \mathbf{E} = \left(\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}\right)$$

$$\Rightarrow \boxed{\nabla^2 \mathbf{E} - \mu_0 \mu_r \varepsilon_0 \varepsilon_r \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0}$$

This is the free-space wave equation

SOLUTION TO THE WAVE EQUATION

For a sinusoidal field, the wave equation reduces to

$$\nabla^2 \mathbf{E} + \frac{\omega^2}{c_r^2}\mathbf{E} = 0$$

The solution to this equation is of the form

$$\mathbf{E} = \mathbf{A}e^{i\left(kr - \omega t + \phi\right)}$$

The speed of light is given by

$$c_r = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}}$$

QUANTUM PROPERTIES OF EM RADIATION

- **Maxwell's equations describe mathematically smooth motion of fields.**
- **For very short wavelengths, it fails to describe certain significant phenomena when the wave interacts with matter.**
- **In those cases, a quantum description is more appropriate.**
- **In this description, the EM radiation is presented by a quantized burst with energy *Q* proportional to the frequency of the wave:**

$$Q = h\upsilon; \quad h = \text{Planck' s constant} = 6.626\times 10^{-34} \text{ Joules/second}$$

- **The energy in the wave train is delivered to a receiver on a probabilistic basis**
- **Only when a large number of wave trains are present, will the overall average effect be described by Maxwell's equations**

WAVE POLARIZATION

From the solution to the wave equation, we can write

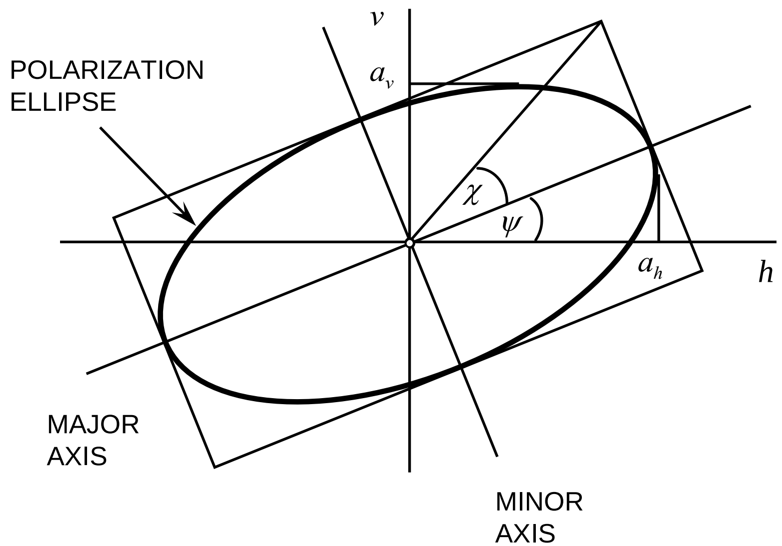
$$\mathbf{A} = A_h e^{i\delta_h}\mathbf{e}_h + A_v e^{i\delta_v}\mathbf{e}_v \Rightarrow \begin{array}{l} E_x = A_x \cos(kz - \omega t + \phi + \delta_h) \\ E_y = A_y \cos(kz - \omega t + \phi + \delta_v) \end{array}$$

This can be written as

$$\left(\frac{E_h}{A_h}\right)^2 + \left(\frac{E_v}{A_v}\right)^2 - 2\frac{E_h}{A_h}\frac{E_v}{A_v}\cos(\delta_h - \delta_v) = \sin^2(\delta_h - \delta_v)$$

This is the expression of an ellipse, and the wave is said to be elliptically polarized

POLARIZATION ELLIPSE



SPECIAL POLARIZATIONS

When

$$\delta = m\pi; \quad m = 0, \pm 1, \pm 2, \ldots$$

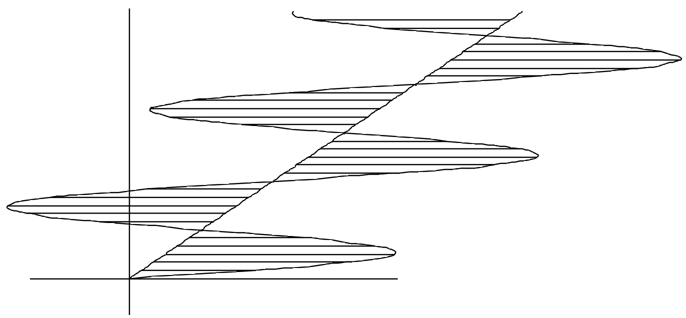
the wave is said to be *linearly* polarized. In this case, the ellipse collapses to form a line.

When

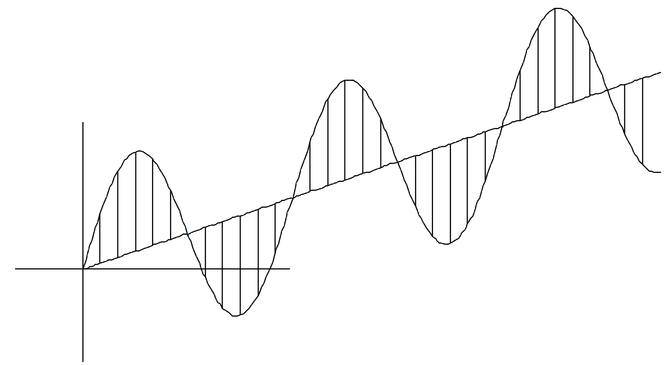
$$A_x = A_y \; and \; \delta = m\pi/2, \quad m = \pm 1, \pm 2, \ldots$$

the wave is said to be *circularly* polarized.

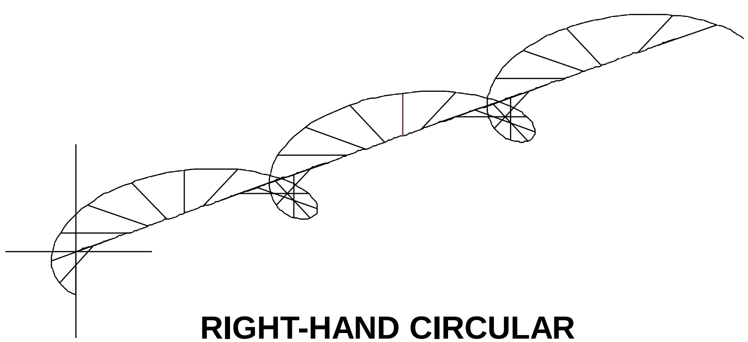
SPECIAL POLARIZATIONS



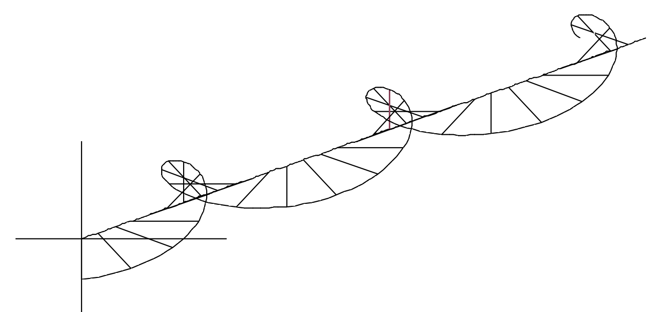
HORIZONTAL (LINEAR)



VERTICAL (LINEAR)



RIGHT-HAND CIRCULAR



LEFT-HAND CIRCULAR

Stokes Parameters

Another way to describe the polarization of a wave, particularly appropriate for the case of partially polarized waves, is through the use of the *Stokes parameters* of the wave. For a monochromatic wave, these four parameters are defined as

$$S_0 = a_h^2 + a_v^2$$
$$S_1 = a_h^2 - a_v^2$$
$$S_2 = 2a_h a_v \cos\left(\delta_h - \delta_v\right)$$
$$S_3 = 2a_h a_v \sin\left(\delta_h - \delta_v\right)$$

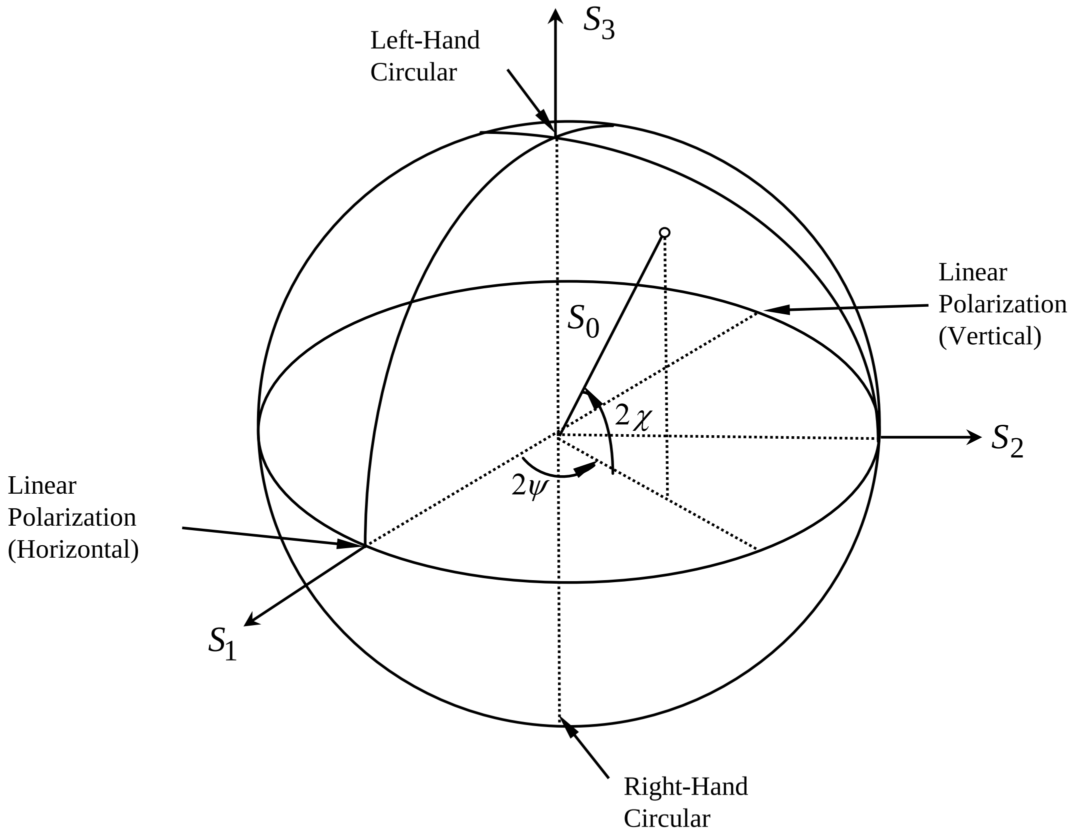
,

Note that for such a fully polarized wave, only three of the Stokes parameters are independent, since $S_0^2 = S_1^2 + S_2^2 + S_3^2$ Using the relations between the ellipse orientation and ellipticity angles and the wave amplitudes and relative phases, it can be shown that the Stokes parameters can also be written as

$$S_1 = S_0 \cos 2\chi \cos 2\psi$$
$$S_2 = S_0 \cos 2\chi \sin 2\psi$$
$$S_3 = S_0 \sin 2\chi$$

These relations lead to a simple geometric interpretation of polarization states. The Stokes parameters can be regarded as the Cartesian coordinates of a point on a sphere, known as the Poincaré sphere, of radius S_0

Poincare Sphere



COHERENCY

- **The** *coherence time* **of two waves of frequency** ν **and** $\nu+\Delta\nu$ **is the time after which the waves are out of phase by exactly one cycle**

$$\nu\Delta t + 1 = (\nu + \Delta\nu)\Delta t \Rightarrow \Delta\nu\Delta t = 1 \Rightarrow \Delta t = \frac{1}{\Delta\nu}$$

- **The** *coherence length* **is defined as:**

$$\Delta l = c\Delta t = \frac{c}{\Delta\nu}$$

- **If two waves are coherent, there is a systematic relationship between their instantaneous amplitudes.**

COHERENCY

- **Assume the total electric field is the sum of two component fields:**

$$E(t) = E_1(t) + E_2(t)$$

- **The average power is**

$$P \sim \langle E^2(t) \rangle = \langle E_1^2(t) \rangle + \langle E_2^2(t) \rangle + 2\langle E_1(t) E_2(t) \rangle$$

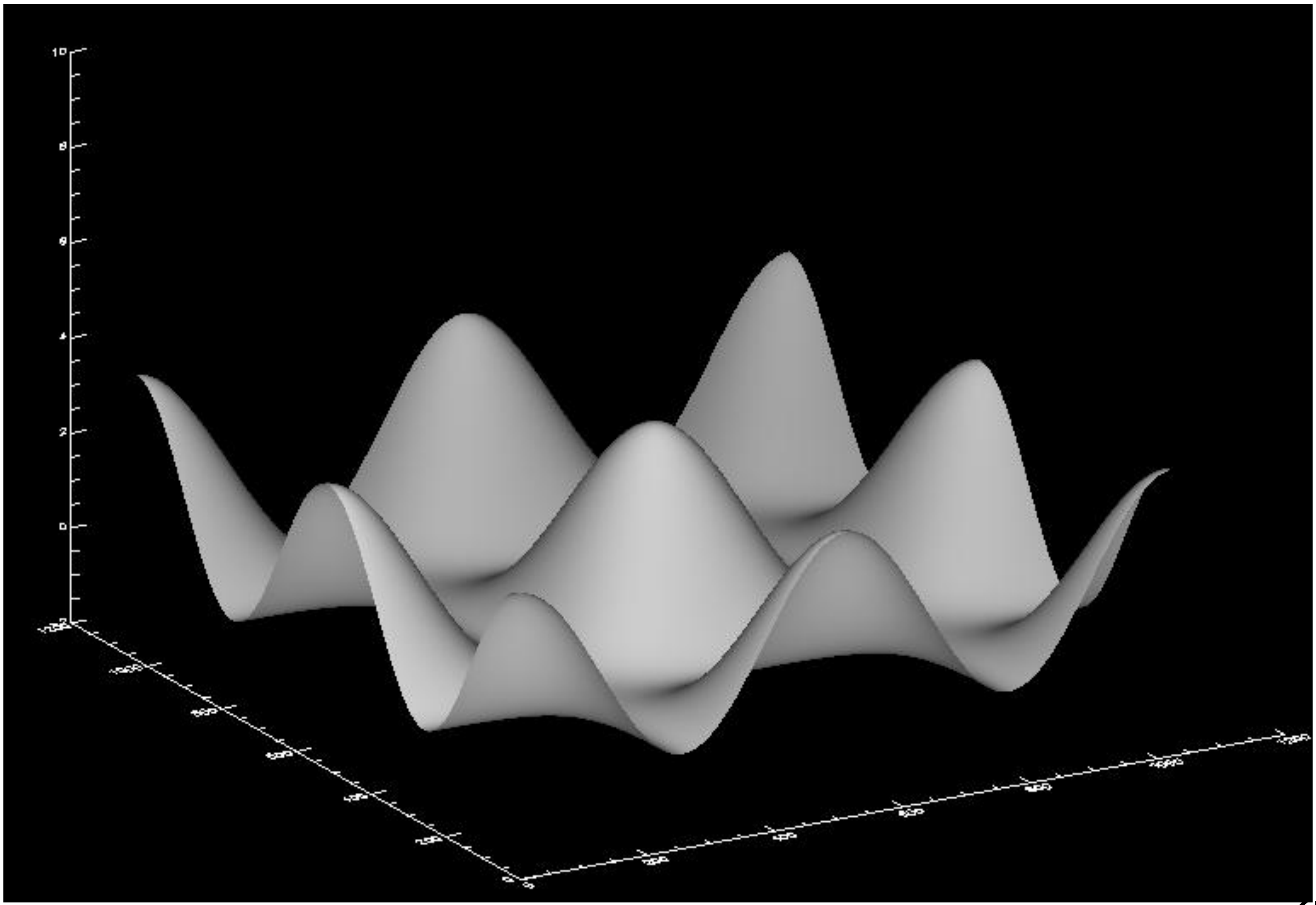
- **If the two waves are incoherent, then**

$$\langle E_1(t) E_2(t) \rangle = 0 \Rightarrow P = P_1 + P_2$$

- **If the waves are coherent, then**

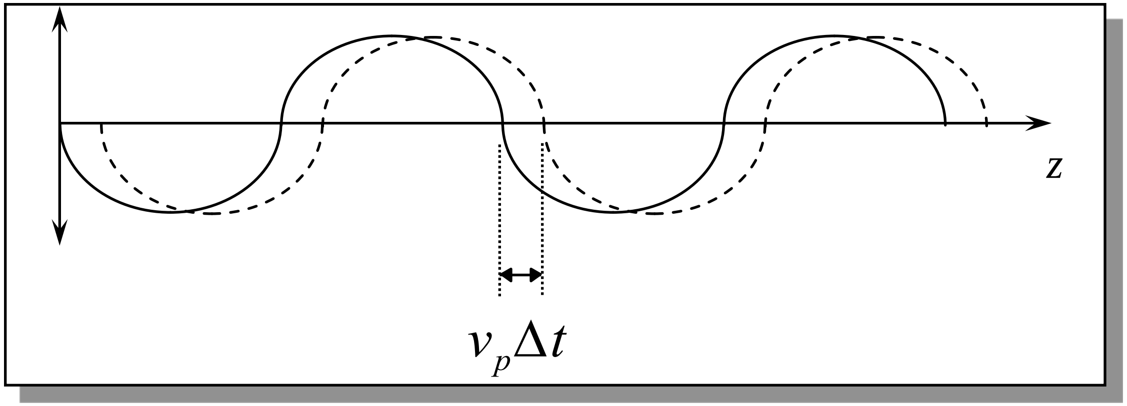
$$\langle E_1(t) E_2(t) \rangle \neq 0 \Rightarrow P > P_1 + P_2 \ or \ P < P_1 + P_2$$

Example of Coherence



PHASE VELOCITY

- **The** *phase velocity* **of a wave is the velocity at which a constant** *phase* **front progresses**



$$\Delta\phi = k\Delta z - \omega\Delta t = 0 \Rightarrow \frac{\Delta z}{\Delta t} = v_p = \frac{\omega}{k}$$

GROUP VELOCITY

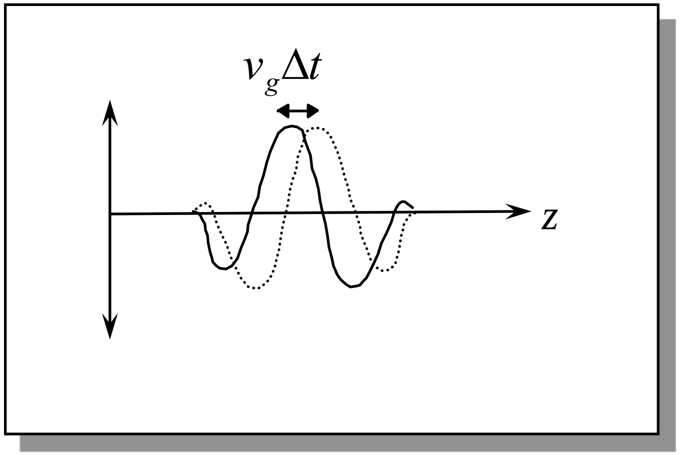
- **The *group velocity* is the velocity at which a plane of constant amplitude progresses**

$$E(z,t) = Ae^{i([k-\Delta k]z-[\omega-\Delta\omega]t)} + Ae^{i([k+\Delta k]z-[\omega+\Delta\omega]t)} = 2Ae^{i(kz-wt)}\cos(\Delta kz - \Delta\omega t)$$

$$\Delta k\Delta z - \Delta\omega\Delta t = 0 \Rightarrow \frac{\Delta z}{\Delta t} = v_g = \frac{\Delta\omega}{\Delta k}$$

- **In the limit, this becomes**

$$v_g = \frac{\partial\omega}{\partial k}$$



PHASE vs GROUP VELOCITY

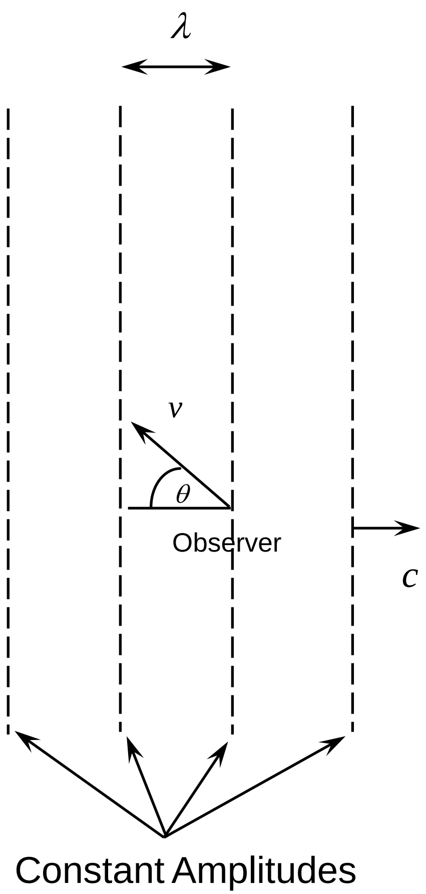
- **Group velocity represents the velocity at which energy is transported by a wave**
- **As such, the group velocity must be less than or equal to the speed of light**
- **For certain media, the phase velocity can be greater than the speed of light**
- **For non-dispersive media, the group and phase velocity are the same and equal to the speed of light**

$$\omega = ck \quad \Rightarrow \quad \begin{cases} v_p = \dfrac{\omega}{k} = c \\ v_g = \dfrac{\partial \omega}{\partial k} = c \end{cases}$$

DOPPLER EFFECT

- **If the relative difference between a source radiating a wave with a fixed frequency ν and an observer changes with time, the frequency of the signal observed will be different than ν**
- **This difference in frequency is known as the Doppler shift**
- **If the distance between the source and the observer is decreasing, the Doppler shift is positive, i.e. the observed frequency is higher than the transmitted one**
- **If the distance is increasing, the Doppler shift is negative**
- **The Doppler shift is used in remote sensing to measure target motion**
- **It is also the effect used in Synthetic Aperture Radar to achieve high resolution in the along-track direction**

DOPPLER EFFECT



$$cT' + vT'\cos\theta = \lambda$$

$$\frac{c}{\nu'} + \frac{v}{\nu'}\cos\theta = \frac{c}{\nu}$$

$$\nu' = \nu + \nu\frac{v}{c}\cos\theta$$

$$\Rightarrow \boxed{\nu_d = \nu\frac{v}{c}\cos\theta}$$

For radars:

$$\boxed{\nu_d = 2\nu\frac{v}{c}\cos\theta}$$

RADIANT ENERGY

- **Radiant energy is the energy carried by the electromagnetic wave**
- **The amount of energy per unit volume is called radiant energy density**
- **Radiant energy** Q **is measured in** $joule$
- **Radiant energy density** W **is measured in** $joule/m^3$

$$W = \frac{dQ}{dV}$$

RADIANT FLUX

- **Radiant flux is the time rate at which radiant energy passes a certain location**

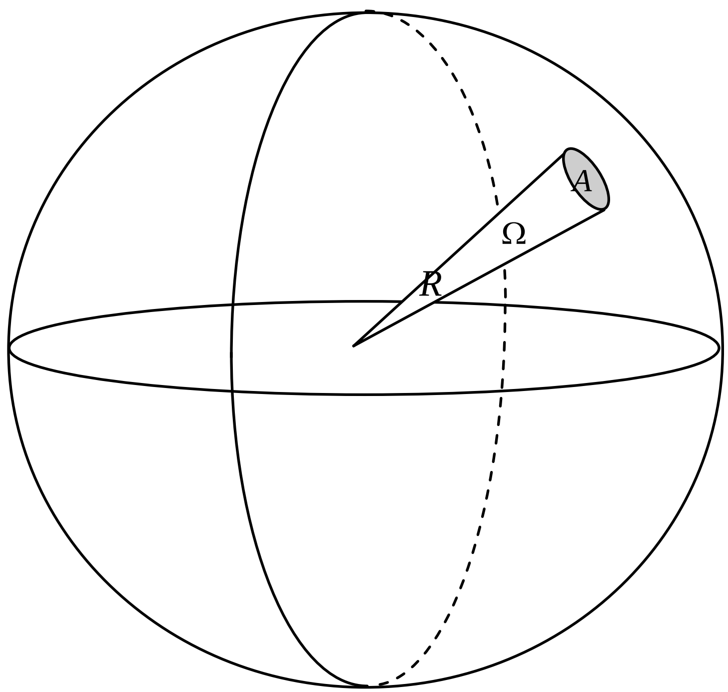
$$\Phi = \frac{dQ}{dt} \quad [watt]$$

- **Radiant flux density is the radiant flux intercepted by a unit area of a plane surface**
- **The flux density** *incident* **upon a surface is called** *irradiance,* M
- **The flux density** *leaving* **a surface is called** *emittance,* E

$$E, M = \frac{d\Phi}{dA} \quad \left[watt/m^2\right]$$

SOLID ANGLE

- **The solid angle Ω subtended by an area A on a spherical surface is that area divided by the radius of the sphere squared**



Area of Sphere

$$= 4\pi R^2$$

$$\Omega = \frac{A}{R^2}$$

RADIANT INTENSITY

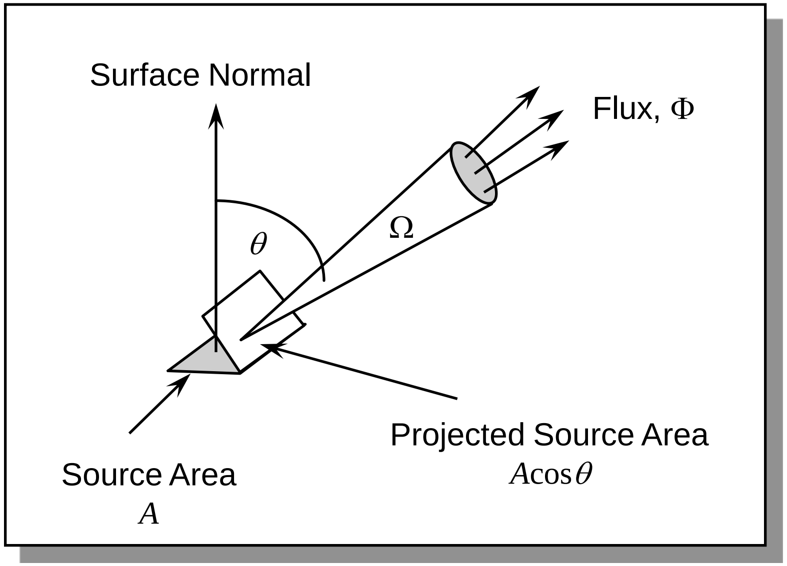
- **The radiant intensity of a point source in a given direction is the radiant flux per unit solid angle leaving the source in that direction**

$$I = \frac{d\Phi}{d\Omega} \quad [watt/steradian]$$

RADIANCE

- **Radiance is the radiant flux per unit solid angle leaving an extended source in a given direction per unit projected area in that direction**

$$L = \frac{dI}{dA\cos\theta} \quad \left[watt / steradian\, m^2\right]$$



- **If the radiance does not change as a function of direction of emission, the source is called** ***Lambertian***

REFLECTANCE, TRANSMITTANCE and ABSORPTANCE

- **Reflectance ρ is the ratio of the reflected exitance from a plane of material to the irradiance on that plane**

- **Transmittance τ is the ratio of the transmitted exitance, leaving the opposite side of the plane, to the irradiance**

- **Absorptance α is the flux density that is absorped over the irradiance**

$$\rho + \tau + \alpha = 1$$

SPECTRAL QUANTITIES

- **Electromagnetic waves are usually made up of a collection of sinusiods of slightly different frequencies, each carrying a part of the radiant flux of the total wave**
- **The spectral band over which these components extend is called the bandwidth of the signal**
- **All radiance quantities have equivalent spectral quantities that correspond to the density as a function of frequency**

$$\text{Spectral flux} = \Phi(\lambda) = \frac{F\text{lux in waves in band } \lambda - \Delta\lambda \text{ to } \lambda + \Delta\lambda}{2\Delta\lambda}$$

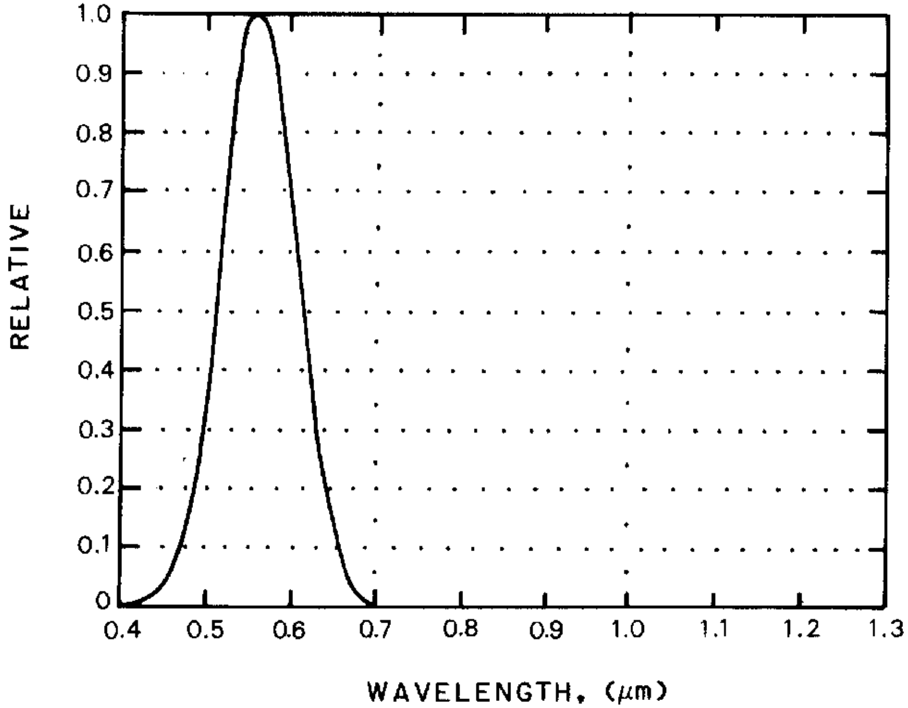
$$\text{Total flux in Bandwidth} = \Phi(\lambda_1 \; to \; \lambda_2) = \int_{\lambda_1}^{\lambda_2} \Phi(\lambda) d\lambda$$

LUMINOUS QUANTITIES

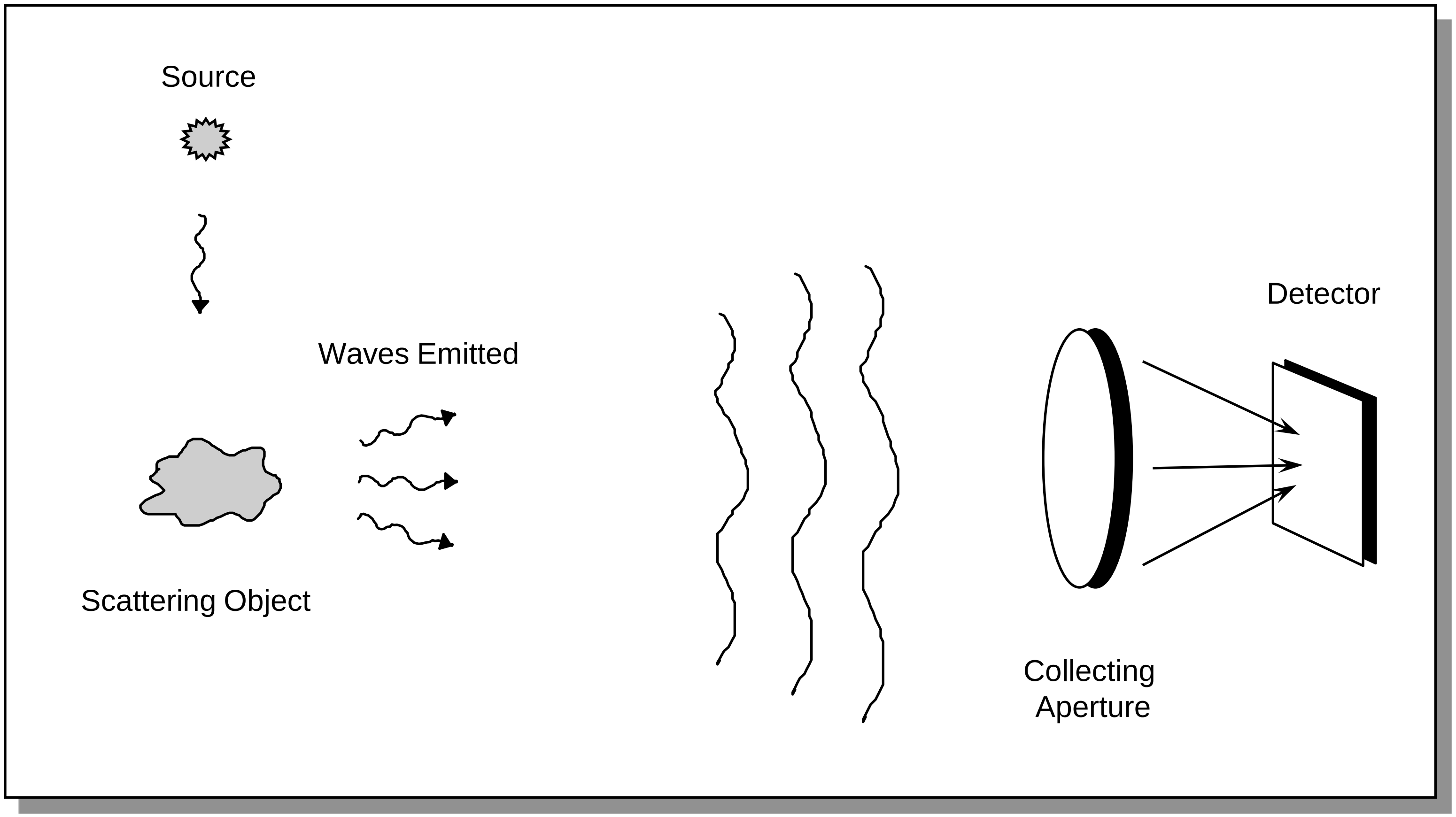
- **Luminous quantities are related to the characteristic of the human eye to perceive radiative quantities**
- **The relative effectiveness of the eye in converting radiant flux to visual response is called the** *spectral luminous efficiency* $V(\lambda)$
- **This function is used as a weighting function in relating radiant quantities to luminous quantities**

$$\Phi_v = 680\int_0^\infty \Phi_e(\lambda)V(\lambda)d\lambda$$

SPECTRAL LUMINOUS EFFICIENCY



COMPONENTS OF A REMOTE SENSING SYSTEM



GENERATION OF EM RADIATION

- A variety of techniques are used to generate electromagnetic radiation in the different parts of the EM spectrum
- At radio frequencies, waves are generated by alternating currents in wires, electron beams, or on the surfaces of antennas
- At microwave frequencies, electron tubes (e.g. TWTs) or molecular exitation (e.g. masers) are used
- In the infrared and visible, waves are generated by molecular excitation (vibrational or rotational) followed by decay. The frequency of the waves generated is exactly related to the difference between the two energy levels of the molecules
- Lasers use the exitation of molecules and atoms and selective decay to generate narrow bandwidth EM radiation, and are used from the UV to the high submillimeter

GENERATION OF EM RADIATION

- **Molecules in a gaseous state tend to have narrow, well-defined emission lines**
- **In the solid phase, the close packing of atoms or molecules distort their electron orbits, leading to a large number of characteristic frequencies**
- **In the case of liquids, the situation is further complicated by the random motion of molecules relative to each other**
- **At higher energies, gamma rays are generated in the natural environment by radioactive decay of uranium, thorium or potassium.**

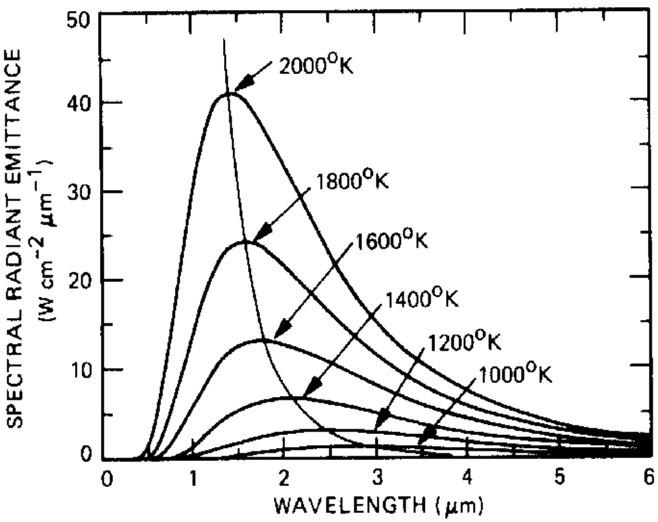
GENERATION OF EM RADIATION

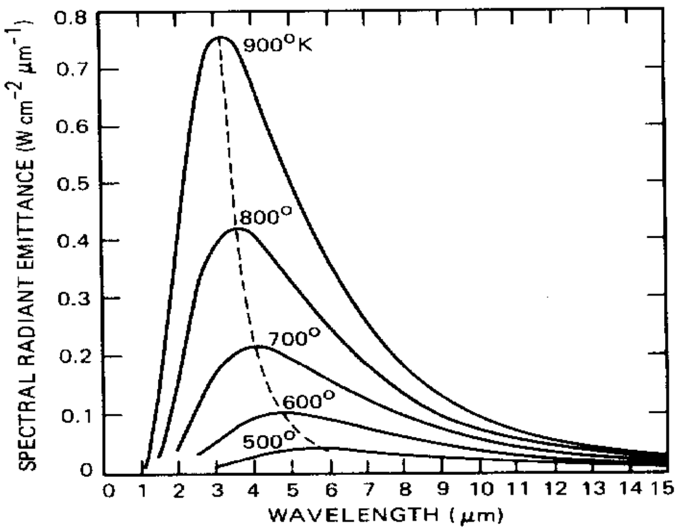
- **Heat energy is a special case of EM radiation**
- **The random motion (due to collisions) of the molecules due to kinetic energy results in exitation (electronic, vibrational and rotational) followed by random emissions during decay**
- **This leads to radiation over a large bandwidth according to Planck's law for an ideal source (called a black body)**

$$S(\lambda) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{ch/\lambda kT} - 1}$$

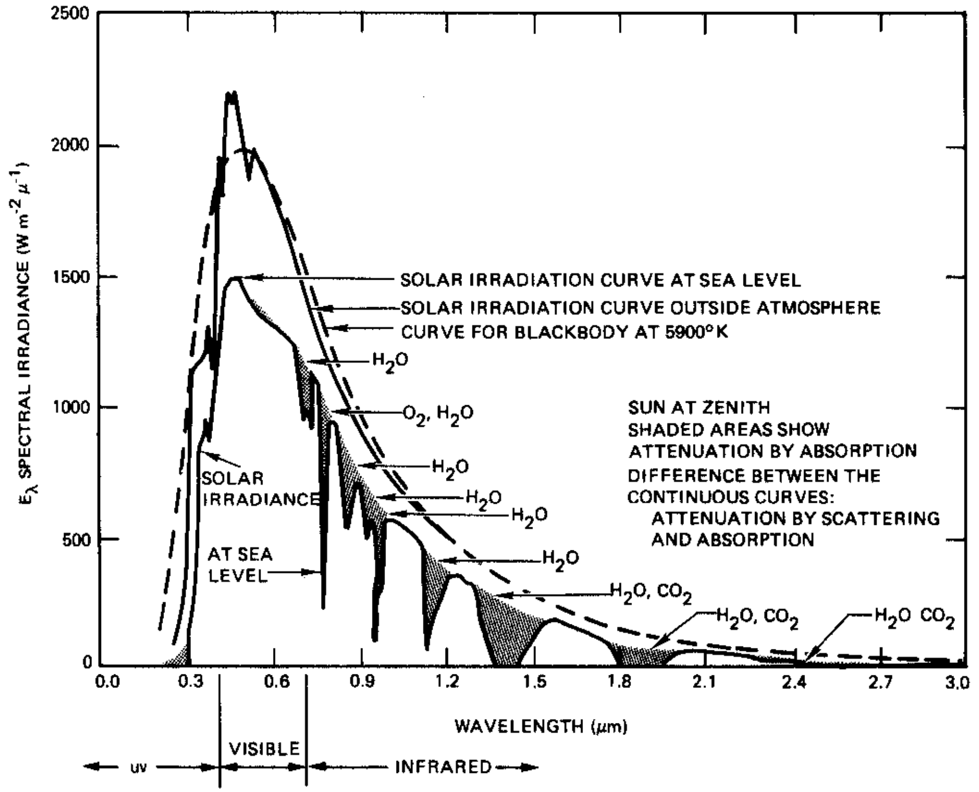
- **Thermal emission is usually unpolarized**

IDEAL BLACK BODY RADIATION





SUN SPECTRAL IRRADIANCE AT EARTH'S SURFACE



DETECTION OF EM RADIATION

- Any remote sensing system uses a collector, followed by a detector, to measure the radiation from the source to be studied.
- The collector is an aperture that intercepts part of the radiated field.
- In the radio and microwave regions of the spectrum, antennas (dipoles, arrays, dishes) are used as collectors.
- In the IR, visible and UV regions, the collector is usually a lens or reflecting surface focussing the energy onto the detector. The energy is then transformed into another form such as heat, electric current, or state change.
- Types of detectors include photomultiplier tubes, photodiodes, and charge coupled devices (CCDs).

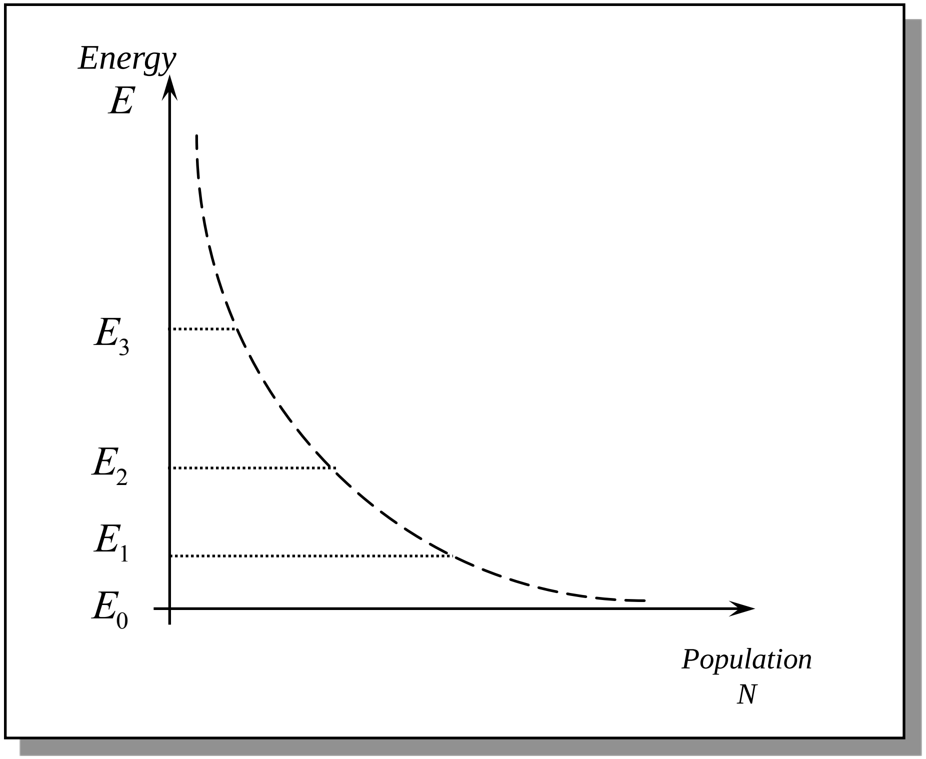
BOLTZMANN’S LAW

- **In the case of thermal equilibrium, the density of population at any energy level is proportional to**

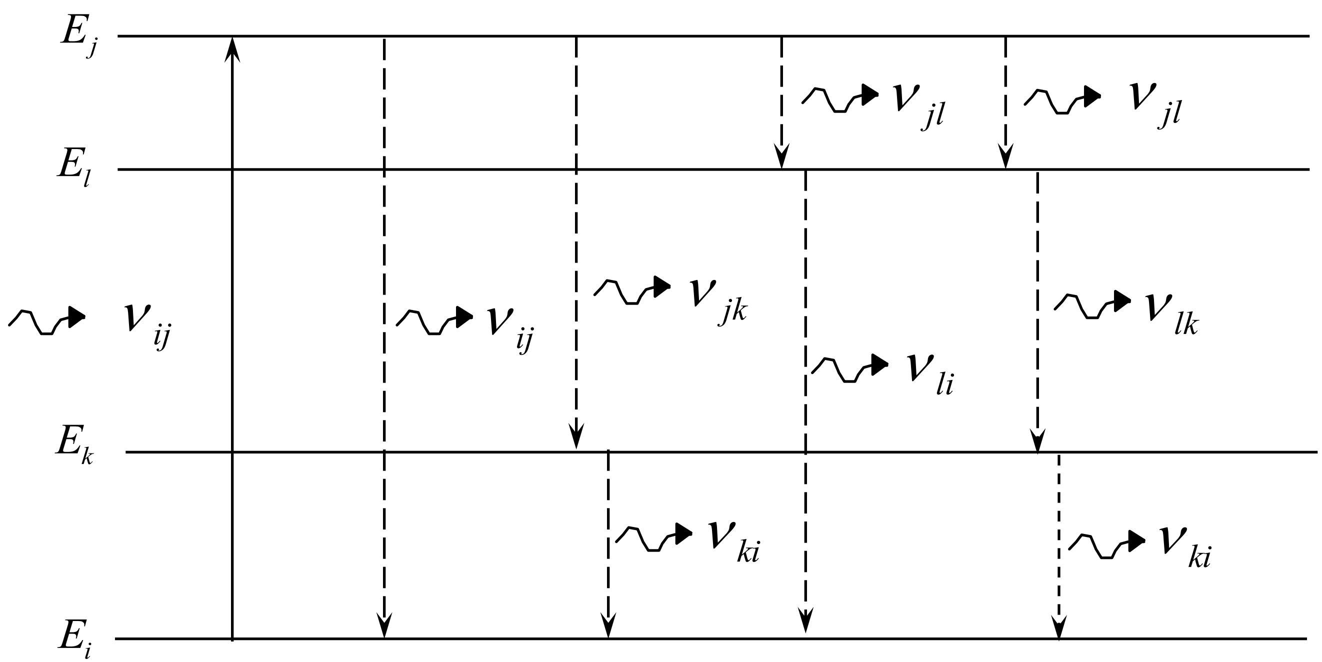
$$N_i \sim e^{-E_i / kT}$$

- **The energy required to excite from level i to level j is**

$$h\nu = E_j - E_i$$



ABSORPTION AND EMISSION



WAVE-MATTER INTERACTIONS

Spectral Region	Main Interaction Mechanisms	Example Applications
Gamma Rays, X-Rays	Atomic Processes	Mapping radioactive materials
Ultraviolet	Electronic Processes	Presence of H and He in atmospheres
Visible and Near IR	Electronic and Vibrational Molecular Processes	Surface chemical composition, vegetation cover, and biological properties
Mid - IR	Vibrational, Vibrational-Rotational molecular processes	Surface chemical composition, atmospheric chemical composition
Thermal IR	Thermal Emission, Vibrational and Rotational Processes	Surface heat capacity, surface temperature, atmospheric temperature, atmospheric and surface constituents
Microwave	Rotational Processes, Thermal Emission, Scattering, Conduction	Atmospheric constituents, surface temperature, surface physical properties, atmospheric precipitation
Radio Frequency	Scattering, Conduction, Ionospheric Effects	Surface physical properties, subsurface sounding, ionospheric sounding

WAVE-MATTER INTERACTIONS