EE/Ae 157a
MIDTERM/FINAL

Due Date: Due EOD November 22, 2019

Open book, but no collaboration
No explicit time limit. It should not take more than ~ 3-4 hours to finish this assignment.

Problem 1 (20 points)

A telescope is orbiting Mars at 300 km altitude. The telescope lens diameter is 40 cm, the focal length is 120 cm, and the focal plane is 4 cm wide. The pixels in the detector are 10 microns on a side. Calculate the swath width, size of the pixels on the ground, and the number of pixels across the swath assuming a pushbroom design. Also, calculate the maximum integration time per pixel, and the resulting data volume per day if we acquire 20 images that are 50 km long in the along-track direction.

The instrument is now changed into a spectrometer. The incoming light is dispersed using a grating such that the different colors are separated spatially in a direction orthogonal to the pushbroom line array. Assume that the wavelength region 0.4 microns to 2.4 microns is dispersed over 640 microns in the focal plane. Now assume that we stack 64 line arrays next to each other to cover the dispersed spectrum. Calculate the bandpass of each channel, the dwell time per spectral channel and the resulting data volume if we acquire the same number of images as before.

Solution

Similar to the solution of the third homework, we solve for the swath width using the imaging geometry:

$$\frac{L}{H} = \frac{l}{f} \rightarrow L = \frac{Hl}{f}$$

Here H is the orbit altitude, l is the size of the focal plane, and f is the focal length of the camera. In this case, $H = 300$ km, $l = 4$ cm, and $f = 120$ cm, which gives

$$L = 10 \text{ km}$$

A 10 micron detector will then project to

$$d = 2.5 \text{ m}$$

on the ground. The number of pixels across the swath is

$$N = \frac{40000}{10} = 4000$$

The maximum dwell time per pixel is the time it takes the footprint of the satellite to move the distance equal to the size of a pixel, *i.e.* 5.875 m. For an orbit altitude of 300 km, the satellite velocity is

$$v_s = \sqrt{\frac{g_s R_m^2}{R_m + H}} = 3410.6 \text{ m/s}$$

Here I have used the radius of Mars as 3396 km and the gravitational acceleration on Mars as 3.7 m/s^2. Assuming that the imager points to the surface underneath the spacecraft, the velocity of the footprint is

$$v = v_s \frac{R_m}{R_m + H} = 3133.75 \text{ m/s}$$

For a pixel of 2.5 m, the integration time is

$$\tau_{max} = 0.8\, m\text{sec}$$

If the images are 50 km long in the along-track direction, it means we acquire a total of 20,000 pixels in the along-track direction, and 4000 pixels in the cross-track direction. Therefore, we acquire a total of

$$Data\ Volume = 20 \times 20{,}000 \times 4000 \times 8 = 1.28 \times 10^{10} \text{ bits/day}$$

The bandpass of each spectrometer channel is simply 1/64 of the spectral range. Therefore,

$$\delta b = \frac{2.4 - 0.4}{64} = 0.03125\ \mu m$$

Since all the spectral channels are acquired simultaneously, the dwell time is the same as that of the pushbroom system, *i.e.*

$$\tau_{max} = 0.8\, m\text{sec}$$

The data volume is simply 64 times that of the single channel. Therefore,

$$Data\ Volume = 64 \times 20 \times 20{,}000 \times 4000 \times 8 = 81.92 \times 10^{10} \text{ bits/day}$$

Problem 2 (20 points)

Consider a planet with radius 1900 km, located at a distance of 1.4×10^{9} km from the Sun and 1.25×10^{9} km from the Earth. The average surface temperature of the planet is 70 K. The surface reflectivity is 0.3 across the visible and infrared spectrum. Plot the received energy by a 1 m^2 aperture telescope in Earth orbit as a function of wavelength between 0.1 and 100 microns. Assume that the Sun is a blackbody with a temperature of 6000 K and a radius of 7×10^{5} km. Now let us assume an area of size 100 km x 100 km at the equator consists of an active volcanic caldera with a temperature of 700 K. Can the earth orbiting telescope uniquely detect the presence of the caldera? Can it measure its size and temperature? Explain. (Hint: The planet is rotating while the telescope observes it.)

Solution:

The total power, per unit wavelength, radiated by the Sun is

$$P_{sun} = S(\lambda, 6000K)\, 4\pi R_{sun}^2$$

The power density at the planet is then

$$p_s = \frac{P_{sun}}{4\pi R_{sun-planet}^2} = S(\lambda, 6000K)\left(\frac{R_{sun}}{R_{sun-planet}}\right)^2$$

The power from the sun that is intercepted by the planet is

$$P_p = p_s A_{eff\ planet} = S(\lambda, 6000K)\left(\frac{R_{sun}}{R_{sun-planet}}\right)^2 \pi R_{planet}^2$$

The planet appears as a disk of radius R_{planet} when viewed from the sun. This can be proven by assuming that the energy from the sun arrives at the planet as parallel rays. Integrating over the surface of a hemisphere of radius R_{planet} gives the effective area as that of a disk of radius R_{planet}.

The amount of power reflected towards the earth is the power intercepted by the planet multiplied by the reflectivity. We shall now assume that this power is distributed isotropically over a hemisphere. In that case, the power density at the earth would be

$$p_{earth} = \frac{\rho P_p}{2\pi R_{planet-earth}^2} = \rho S\left(\lambda, 6000K\right)\left(\frac{R_{sun}}{R_{sun-planet}}\right)^2 \frac{\pi R_{planet}^2}{2\pi R_{planet-earth}^2}$$

If the telescope has a collecting area $A_{telescope}$, the power collected will be

$$P_{earth}^{ref} = \frac{1}{2}\rho S\left(\lambda, 6000K\right)\left(\frac{R_{sun}}{R_{sun-planet}}\right)^2 \left(\frac{R_{planet}}{R_{planet-earth}}\right)^2 A_{telescope}$$

We added the superscript *ref* to distinguish the reflected power from that emitted by the planet itself.

Now let us consider the power radiated by the planet. The total power, per unit wavelength, radiated by the planet is

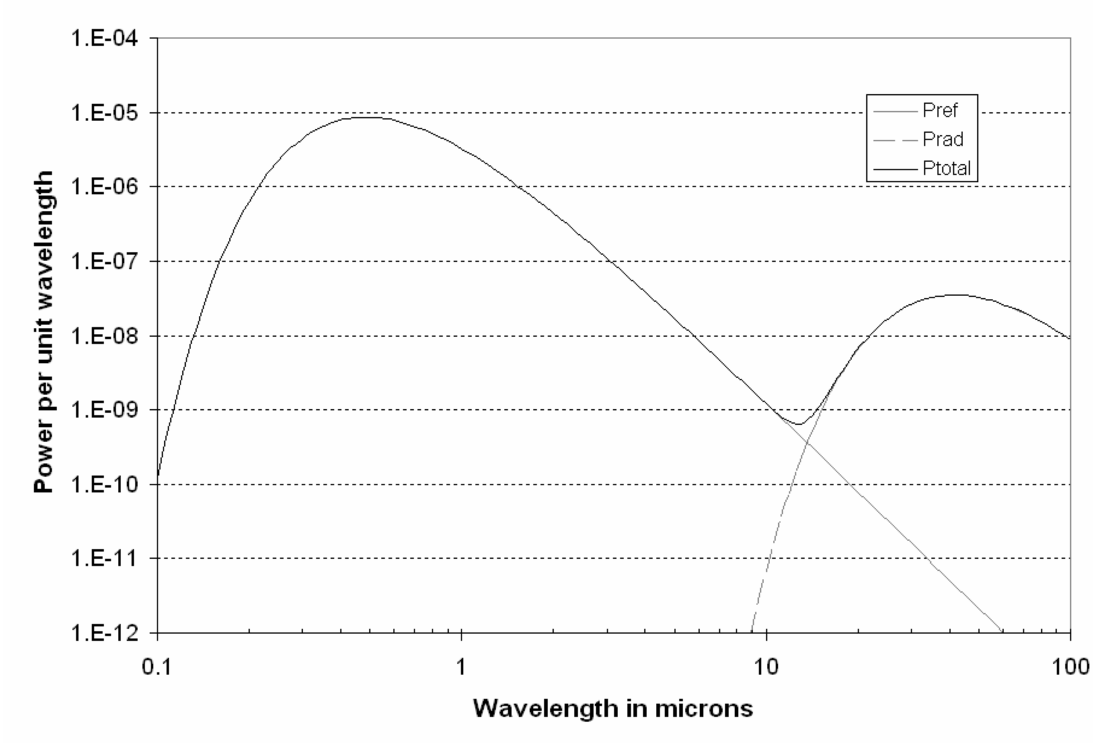
$$P_{planet}^{rad} = \left(1-\rho\right) S\left(\lambda, 70K\right) 4\pi R_{planet}^2$$

This power is distributed uniformly over a sphere. The power received by the telescope will then be

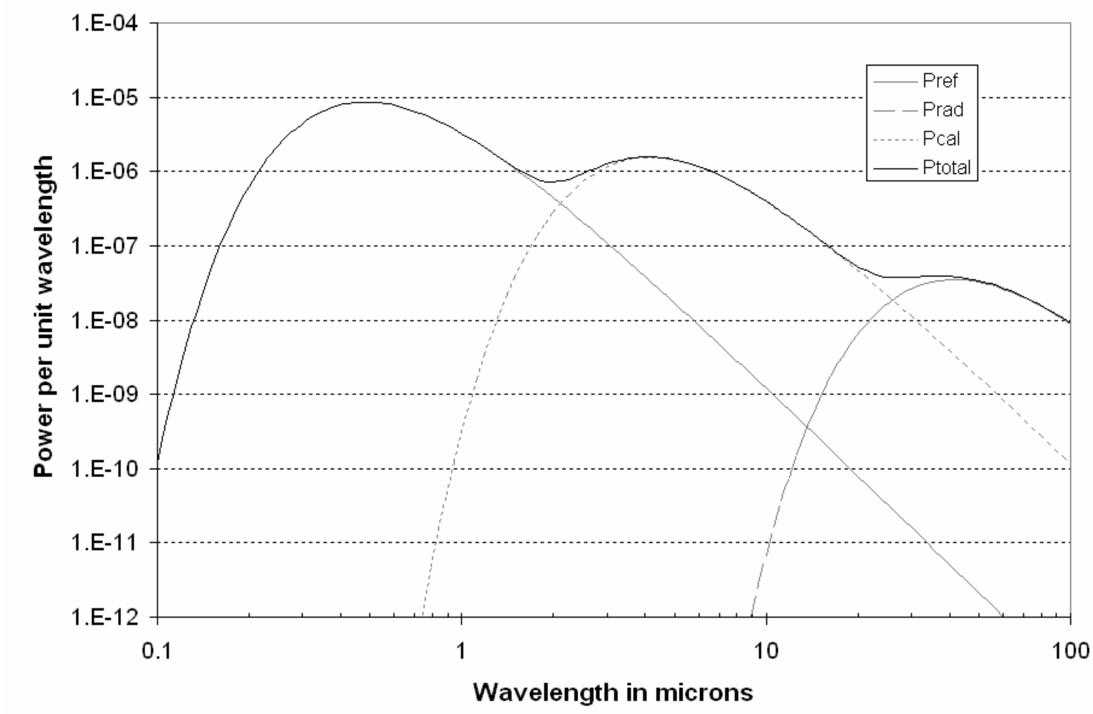
$$P_{earth}^{rad} = \left(1-\rho\right) S\left(\lambda, 70K\right)\left(\frac{R_{planet}}{R_{planet-earth}}\right)^2 A_{telescope}$$

In the absence of additional information, we shall assume that the emittance of the caldera is the same as that of the rest of the planet. Following the same argument as before for the planet itself, we find that the power received by the telescope from the caldera, *when it is completely visible*, is

$$P_{earth}^{ccal} = \left(1-\rho\right) S\left(\lambda, 700K\right)\left(\frac{A_{caldera}}{2\pi R_{planet-earth}^2}\right) A_{telescope}$$



The combined signal from the reflected sunlight, caldera, and planet emission is shown below



A comparison of the two figures clearly shows that it is possible to detect the presence of the caldera with a spectrometer. The position of the maximum of the caldera signal provides the information about the temperature of the caldera through Wien's law. The strength of the signal at that frequency allows one to estimate the combination of the emissivity and the size of the caldera. On might also be able to get an estimate of the size alone, at least in one dimension, by observing the change in the signal at the wavelength where the maximum radiation from the caldera occurs as the planet rotates. As the caldera comes partially into view, one would see the signal change until the entire caldera is visible. The fraction of the planet rotation over which this occurs provides information about the size of the caldera along the equator.

This problem illustrates that even though the spatial resolution of the telescope is inadequate to uniquely image the caldera, we can still infer a lot of information about the caldera if we have sufficient spectral information available.

Problem 3. (10 points)

A five band camera is used to image a scene with a heterogeneous distribution of minerals. From our knowledge of the area being imaged, we expect three minerals with the following spectra to be present in the image:

S1 = [.9 .1 .9 .9 .9]
S2 = [.9 .9 .1 .9 .9]
S3 = [.9 .9 .9 .1 .9]

The following four spectra are extracted from our image:

SM1 = [0.95 0.58 0.60 0.99 0.98]
SM2 = [1.23 0.77 0.94 0.85 1.26]
SM3 = [1.25 1.02 0.45 0.98 1.11]
SM4 = [1.15 1.09 0.45 0.96 1.17]

Calculate the relative abundance of each of the three minerals in these spectra.

Solution:

We postulate the following model for the observations

$$\mathbf{SM} = [\mathbf{M}]\mathbf{A}$$

Here $\mathbf{SM}$ represents the spectral measurement, $[\mathbf{M}]$ is the matrix containing the minerals expected to be present in the scene, and $\mathbf{A}$ is the vector containing the abundances. Multiplying both sides by the transpose of $[\mathbf{M}]$, we find

$$[\mathbf{M}^T]\mathbf{SM} = [\mathbf{M}^T\mathbf{M}]\mathbf{A}$$

Now we multiply both sides by the inverse of the matrix on the right to find the solution

$$\mathbf{A} = [\mathbf{M}^T\mathbf{M}]^{-1}[\mathbf{M}^T]\mathbf{SM}$$

In our case,

$$[\mathbf{M}] = \begin{bmatrix} .9 & .9 & .9 \\ .1 & .9 & .9 \\ .9 & .1 & .9 \\ .9 & .9 & .1 \\ .9 & .9 & .9 \end{bmatrix}$$

Performing the inversion as above, we find:

A1 = [0.55, 0.52, 0.03]
A2 = [0.56, 0.34, 0.46]
A3 = [0.18, 0.89, 0.23]
A4 = [0.11, 0.91, 0.28]

If we convert these to relative abundances, we find

A1 = [0.50, 0.47, 0.03]
A2 = [0.41, 0.25, 0.34]
A3 = [0.14, 0.68, 0.18]
A4 = [0.08, 0.70, 0.22]