EE/Ae 157a

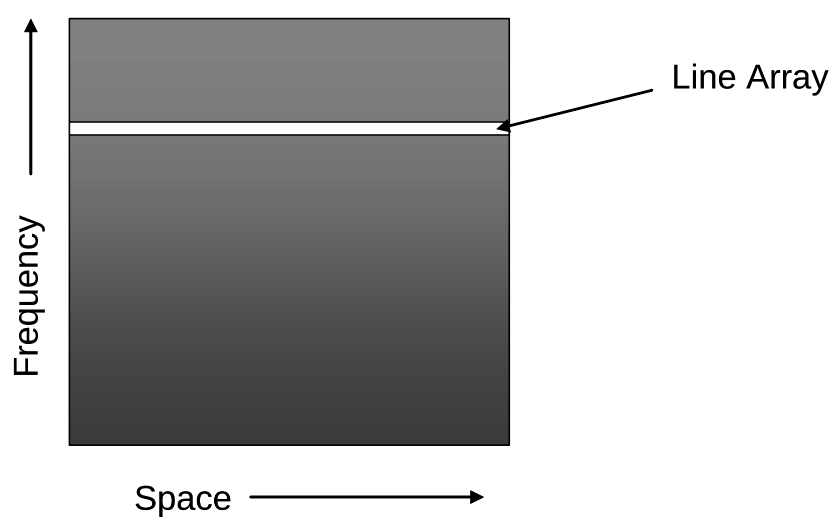
Homework #3
Due Date: Wednesday November 13, 2019

Problem 1 (20 points)

A telescope is orbiting the earth at 705 km altitude. The telescope lens diameter is 40 cm, the focal length is 120 cm, and the square focal plane is 10 cm on a side. The pixels in the detector are 10 microns on a side. Calculate the swath width, size of the pixels on the ground, and the number of pixels across the swath assuming a pushbroom design. Also, calculate the maximum integration time per pixel, and the resulting data rate.

The instrument is now changed into a spectrometer. The incoming light is dispersed using a prism such that the different colors are separated spatially in a direction orthogonal to the pushbroom line array as shown in the figure. To measure the different spectral components, the prism mechanism is scanned so that different colors sweep over the line array. Calculate the scan rate of the dispersion mechanism. If we are to measure 128 spectral channels, what would the integration time be per spectral channel? What is the data rate of the spectrometer?

A different implementation arranges a stationary dispersive element such that the light from 0.4 microns to 0.9 microns fall in the vertical direction on an array that has 128 detector lines stacked on top of each other. What is the bandwidth per channel, the integration time per spectral channel, and the data rate in this case?



Solution:

We find the size of the swath using the imaging geometry:

$$\frac{L}{H} = \frac{l}{f} \quad \Rightarrow \quad L = \frac{Rl}{f}$$

Here H is the orbit altitude, l is the size of the focal plane, and f is the focal length of the camera. In this case, $H = 705$ km, $l = 10$ cm, and $f = 120$ cm, which gives

$$L = 58.75\,\text{km}$$

A 10 micron detector will then project to

$$d = 5.875\,\text{m}$$

on the ground. The number of pixels across the swath is

$$N = \frac{10\ cm}{10\ \mu m} = 10,000$$

The maximum dwell time per pixel is the time it takes the footprint of the satellite to move the distance equal to the size of a pixel, *i.e.* 5.875 m. For an orbit altitude of 705 km, the satellite velocity is

$$v_s = \sqrt{\frac{g_s R_e^2}{R_e + R}} = 7507.3\,\text{m}/\text{sec}$$

Assuming that the imager points to the surface underneath the spacecraft, the velocity of the footprint is

$$v = v_s \frac{R_e}{R_e + R} = 6760.3\, m/\text{sec}$$

For a pixel of 5.875 m, the integration time is

$$\tau_{\max} = 0.869\,\text{msec}$$

To calculate the data rate, we shall assume we use the maximum dwell time per pixel. We then acquire 10,000 data samples every 0.869 msec. Assuming that each sample is coded as 8 bits, we find the data rate to be

$$D = \frac{10000}{0.000869} \times 8 = 87.79 \text{ Mbits/sec}$$

In the case of the spectrometer, we have to complete all 128 spectral measurements in the time we could make one measurement before. The prism must therefore complete one scan every 0.869 msec. But we can scan the frequencies in increasing order once, followed by decreasing order next. So, we can really use twice this time per scan. The scan rate is then

$$\text{Scan Rate} = \frac{1}{2 \times 0.000869} = 575.3 \text{ Hz}$$

The integration time per spectral channel is 1/128 th of the previous integration time

$$\tau_s = 6.79 \ \mu\text{sec}$$

The data rate is simply 128 times higher than before, since we are making 128 measurements in the time it took to make one before. Therefore

$$D_s = 11.2 \text{ Gbits/sec}$$

For the case where we use a detector array with a stationary dispersive element, the bandwidth per channel will be

$$Bandwidth = \frac{(0.9 - 0.4)}{128} \mu\text{m} = 3.9 \text{ nm}$$

The integration time is the same is that of the original pushbroom imager, because we image all the spectral channels at the same time.

The data rate is the same as the previous case. The difference between the two implementations is that we use a 2-dimensional array of detectors to eliminate the need for scanning the dispersive element and gain back the integration time. Note that because of the difference in integration time, we will collect 128 times more photons per pixel in the second case. This means the images of the second case will have much better signal to noise ratio that the case of the scanning mechanism. We also do not have moving parts in the second case.