

EE/Ae 157a

Homework #2

Due Date: Wednesday October 30, 2019

Bring to class or place in the mailbox in Moore Building

Problem 1 (20 points)

A telescope is orbiting the earth at 705 km altitude. The telescope lens diameter is 40 cm, the focal length is 120 cm, and the square focal plane is 20.916 cm on a side. The solar power density at the surface of the earth is 1.37 kW/m^2 . Ignore losses in the atmosphere.

- (a) Calculate the power density at the lens if the earth albedo is 50%. Calculate this power density per unit area in the scene, *i.e.* assume that we collect the reflected power from a $1\text{m} \times 1\text{m}$ area in the scene.
- (b) Calculate the total *energy* intercepted from a $30\text{m} \times 30\text{m}$ ground pixel if the shutter stays open for 0.1 millisecond. (Hint: Energy is power \times integration time)
- (c) Calculate the field-of-view and the swath width of the telescope.
- (d) If the instrument is a framing camera, calculate the number of detectors required to acquire a square image with each side as large as the swath width filled with $30\text{m} \times 30\text{m}$ pixels on the ground. Also, calculate the integration time if we assume that we use a scanning mirror to stare at the same area with a 75% duty cycle. This means we integrate the signals for 75% of the time it takes the projection of the focal plane on the ground to move the distance equal to the total size of the image in the along-track dimension.
- (e) In the pushbroom configuration with a single line of detectors, calculate the number of detectors required for a pixel size of $30\text{m} \times 30\text{m}$ on the ground to cover the entire swath. Also calculate the integration time and total energy per pixel assuming that image smearing is to be minimized by limiting the integration time to $1/3$ of the total possible integration time for a pixel.
- (f) For a scanning mirror system (wiskbroom configuration), calculate the mirror scan rate, the integration time per pixel, and the total energy intercepted from each pixel assuming that we use an array of 16 detectors arranged in the along-track direction.
- (g) Now assume that we place a filter in the focal plane with a 1% transmission bandwidth centered at 0.5 microns, *i.e.* only 1% of the incoming energy passes through the filter. Calculate the number of photons received from a pixel for each of the three cases above.
- (h) Calculate the data rate in bits/second for each of the three cases, assuming we digitize the data from each pixel to 8 bits.

Solution:

- (a) Since the power density at the surface of the earth is $p_i = 1.37 \text{ kW/m}^2$, the total energy reflected by 1 m^2 with an albedo $\omega = 0.5$ is

$$P_r = p_i \omega$$

Using the supplied values, we find

$$P_r = 685 \text{ W}$$

This power is reflected isotropically over half a hemisphere, *i.e.* 2π steradians. Since the power is spread isotropically over the surface of the hemisphere, the energy density a distance R away from the ground element is

$$M = \frac{P_r}{2\pi R^2} = \frac{p_i \omega}{2\pi R^2}$$

For $R = 705 \text{ km}$, we find

$$M = 219.3 \times 10^{-12} \text{ W/m}^2$$

- (b) The total area of the lens is

$$A = \frac{\pi}{4} d^2 = 125.7 \times 10^{-3} \text{ m}^2$$

The lens captures all the power reflected by an area S of $30\text{m} \times 30\text{m}$ and focuses that on a detector. The total power captured by the lens is

$$P_c = MSA = \frac{p_i \omega SA}{2\pi R^2} = 24.81 \times 10^{-9} \text{ W} \quad (\text{or } J/\text{sec})$$

If the lens stays open for a time $\tau = 0.1 \text{ msec}$, the total energy per detector element is

$$E = P_c \tau = \frac{p_i \omega SA \tau}{2\pi R^2} = 2.48 \times 10^{-12} \text{ J}$$

- (c) The field of view (FOV) of the camera is the angle subtended by the size of the focal plane relative to the focal length. From the figure on the right, we find that

$$FOV = 2 \tan^{-1} \left(\frac{l/2}{f} \right)$$

where l is the size of the focal plane and f is the focal length. Using the values $l = 20.916 \text{ cm}$ and $f = 120 \text{ cm}$ we find

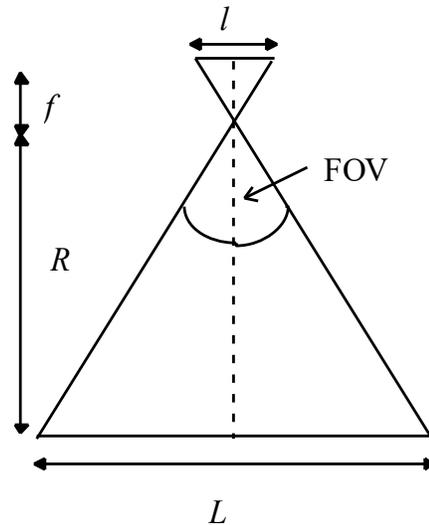
$$FOV = 9.96^\circ = .174 \text{ radians}$$

The swath width L of the instrument is calculated from the figure on the right as

$$\frac{L}{R} = \frac{l}{f} \Rightarrow L = \frac{Rl}{f}$$

Using the numerical values for this instrument, we find a swath width of

$$L = 122.9 \text{ km}$$



- (d) This instrument has a ground resolution of S in area. Therefore, each pixel has a size \sqrt{S} on a side. The number of pixels needed to cover the swath is therefore

$$n = \frac{L}{\sqrt{S}} = 4096$$

If the framing camera must acquire a square image, we will need 4096 x 4096 pixels.

If the satellite footprint moves at velocity v , the maximum dwell time is

$$\tau = n \frac{\sqrt{S}}{v}$$

This is a subtle, but important point. The satellite footprint is the spot on the surface of the earth, and that is where we want overlap between pixels, not at the altitude of the spacecraft. For an orbit altitude of 705 km, the satellite velocity is

$$v_s = \sqrt{\frac{g_s R_e^2}{R_e + R}} = 7507.3 \text{ m/sec}$$

Assuming that the imager points to the surface underneath the spacecraft, the velocity of the footprint is

$$v = v_s \frac{R_e}{R_e + R} = 6760.3 \text{ m/sec}$$

The scan mirror allows us to use 75% of the maximum dwell time. The useable dwell time therefore is

$$\tau = 13.63 \text{ sec}$$

The corresponding available energy is (see part (b))

$$E = P_c \tau = 0.338 \times 10^{-6} \text{ J}$$

- (e) The pushbroom line array still has to cover the entire swath at a time. Therefore, we need 4096 pixels in the line array. To keep smearing to a minimum, we only use 1/3 of the maximum integration time. The useable integration time therefore is (same as in part (d)), but with $n=1$ since we use a single line array)

$$\tau = 1.48 \text{ msec}$$

The corresponding available energy is (see part (b))

$$E = P_c \tau = 36.7 \times 10^{-12} \text{ J}$$

- (f) The mirror has to complete one scan in the time it takes the satellite footprint to move a distance equal to the size of the 16 pixels in the along-track direction, *i.e.* $16\sqrt{S}$. The rate of oscillation of the mirror is therefore

$$r = \frac{v}{16\sqrt{S}} = 14.1 \text{ Hz}$$

Let us assume that half the scan is devoted to imaging. (The other half of the scan is used to image in the opposite direction.) This time is to be divided among n detectors. Therefore, the dwell time is

$$\tau = \frac{1}{2n} \frac{16\sqrt{S}}{v} = 8.67 \mu\text{sec}$$

The energy per pixel is

$$E = P_c \tau = 0.215 \times 10^{-12} \text{ J}$$

(g) If the filter has a relative bandwidth B , only a fraction B of the energy is absorbed. The energy per photon at 0.5 microns is

$$E = hc/\lambda = 3.976 \times 10^{-19} \text{ J}$$

Using $B = 0.01$, we find

Imager Type	Energy in Joules	Photons @ 0.5 microns
Framing Camera	0.338×10^{-8}	8.5×10^9
Pushbroom	36.7×10^{-14}	923030
Scanning Mirror	0.215×10^{-14}	5408

(h) Calculate the data rate for each of the three cases, assuming we digitize the data from each pixel to 8 bits.

The data rate for the three cases is identical, since in all cases we image the equivalent of a 4Kx4K image while the satellite moves the distance of one frame. The data rate is

$$D = \frac{N \times N \times 8}{T_f} = 7.38 \text{ MBits / sec}$$