EE/Ae 157a

Homework #1

Due Date: October 21, 2019

Problem 1.

Calculate the altitude of geostationary orbits for Mercury, Venus, Mars, Jupiter and Saturn. The rotation periods of these planets are 58.7 Earth-days, 243 Earth-days, 24 hr 37 min, 9 hr 51 min, and 10 hr 14 min, respective1y.

Solution:

For a geostationary orbit, the orbit period is the same as the rotation period of the planet. Therefore, from equation B-5, the orbit altitude is

$$h = \left[\frac{g_s R^2 T^2}{4\pi^2} \right]^{1/3} - R$$

Where T is the rotational period of the planet. The results are

Planet	Radius R (km)	Surface Gravity g_s (m/sec^2)	Rotation Period (sec)	Geostationary Orbit Altitude (km)
Mercury	2440	3.63	5057827	238598
Venus	6050	8.83	20937852	1525009
Earth	6380	9.81	86164	35809
Mars	3395	3.92	88620	17397
Jupiter	71,500	25.9	35460	90064
Saturn	60,000	11.38	36840	52092

Problem 2.

Let us assume that an Earth orbiting sensor in sun-synchronous circular orbit requires daily repeat (*i.e.* 1 solar day). Calculate the lowest three orbits which allow such a repeat coverage. Calculate the lowest three orbits for repeats every two solar days (N = 2) and every three solar days (N = 3).

Solution:

This problem is similar to the previous one, except that the period of the orbit is different. To find the lowest three altitudes, we need to select the number of orbits in the repeat period. We shall assume that we want the orbit altitude to be larger than 200 km. The results are shown below.

Repeat Period in Days	Number of Orbits in Period	Altitude in km
1	16	264.4
1	15	556.5
1	14	883.0
2	31	406.5
2	29	715.0
2	27	1061.2
3	47	358.3
3	46	455.6
3	44	661.2

Problem 3.

A radar mission is designed to fly on the space shuttle with the aim of mapping as much of the Earth as possible in 10 days. Given the size and mass of the radar payload, the inclination of the orbit is 57°, and the altitude range is 200 km to 250 km. Calculate the altitude of an orbit in this range that would repeat in approximately 10 siderial days, taking orbit precession into account. Calculate the separation between orbit tracks along the equator.

Solution:

In this case, we know the inclination, so we have to pick the orbital altitude such that we get an exact repeat after N orbits. From Problem 2 we know that 16 orbits in 1 day leads to about 264 km orbit if we are in a sun-synchronous orbit. Here we have a different inclination, but we can start by picking N to be close to 160. Then we can iterate the altitude to find the exact orbit altitude that will complete that many orbits in approximately 10 days.

Using the expressions in the Orbit Mechanics slides, we know we have to find the altitude that would ensure that

$$T_r = \frac{MP_e}{\left(1 - \frac{\dot{\Omega}}{2\pi} P_e\right)} = NP_o$$

Iterating this equation, we find the optimum altitude for 159 orbits to be 233.2 km. We can also try 161 orbits, which requires an orbit altitude of 176.6 km. This is clearly too low.

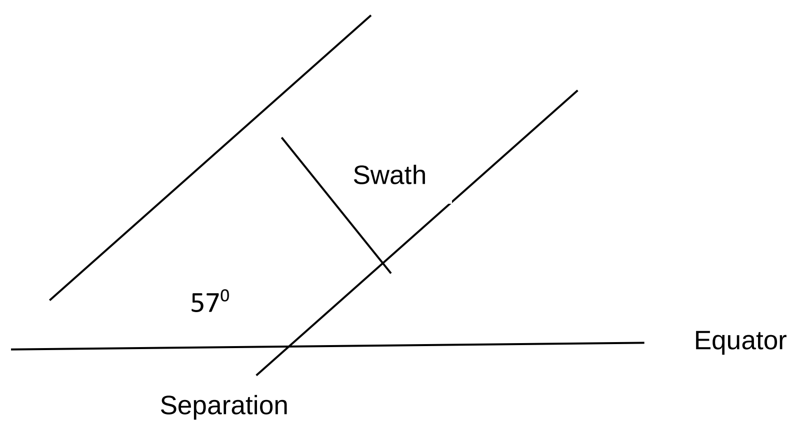
The circumference of the earth at the equator is

$$C = 2\pi R = 40086.7 \text{ km}$$

The separation between orbits will then be

$$S = \frac{2\pi R}{N} = 252.1 \text{ km}$$

The orbit crosses the equator at an angle of 57^0 as shown in the drawing below.



The minimum swath width required to cover the 252 km separation between orbits is therefore

$$Swath = S \times \sin(i) = 211.4 \text{ km}$$

In practice we will use a larger value than this by ~10% to account for altitude drifts and local topography.

An important point to remember is that when we pick the number of orbits to complete in the repeat cycle, we should pick an integer that will provide that number of *unique ground tracks* during the repeat cycle. As an example, consider the case of 158 orbits for this problem. The orbital altitude of that choice is 261.9 km. Should be a valid orbit, right? The issue with this orbit is that 158/10 = 79/5, which means that this choice will indeed have the satellite orbit 158 times around the earth in the 10 days, but the 158orbits are really two sets of 79 *ground tracks* that repeat in 5 day intervals. One could still use this orbit, but now we will require twice as wide a swath in order to cover the equator.

Problem 4.

The first client of your satellite company wants a measurement of the surface of the earth between 60 degrees north and 60 degrees south every three solar days, *i.e.* the orbit needs to be sun-synchronous. The instrument can cover a field of view equal to 30 degrees. For cost reasons, you decide that you can only fly the instrument between 400 km and 800 km altitudes.

- Calculate the possible orbits altitudes that would satisfy the 3-day repeat requirement.
- Compare the instrument swath width to the required swath to achieve global coverage for each orbit. Can any of these orbits provide global coverage with a single satellite?
- If the orbits cannot provide global coverage with a single satellite, how many satellites would you need to provide the global coverage?

Solution:

Note: This problem statement is inconsistent. If we only cover the earth between 60 N and 60 S, *i.e.* we use an inclination of 60 degrees, then the orbit cannot be sun-synchronous. Here I will provide the solution for a 60 degree inclination.

Since the coverage has to be between 60 degrees north and 60 degrees south, we need an inclination of about 60 degrees. Strictly speaking the imager will cover more than this because of the field of view, but for now we shall assume 60 degrees. The orbits that give exact 3-day repeats are:

Repeat Period	Number of Orbits	Altitude in km	Required Swath	Instrument Swath
3	46	404.6	755	217
3	44	613.9	789	330
3	43	724.4	807	390

To find width of the instrument swath, we need to know the angular width of the swath as seen from the center of the earth. Consider the triangle formed by the satellite, the center of the earth, and the edge of the swath as shown in Figure 4.1.

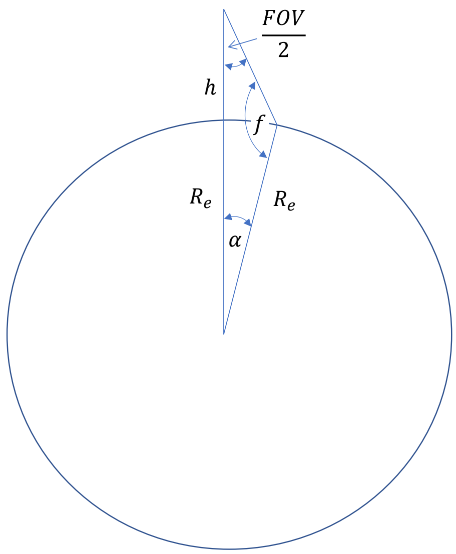


Figure 4.1. Geometry for calculating the swath width from the instrument field of view.

From the law of sines we can write

$$\frac{\sin(FOV/2)}{R_e} = \frac{\sin(f)}{R_e + h} \rightarrow \sin(f) = \sin(FOV/2)\frac{R_e + h}{R_e}$$

But using the fact that all the angles in a triangle adds to 180 degrees, we find,

$$\sin f = \sin(180 - \alpha - FOV/2) = \sin(\alpha + FOV/2)$$

Using this in the previous equation, we find

$$\alpha = \sin^{-1}\left\{\frac{R_e + h}{R_e}\sin(FOV/2)\right\} - FOV/2$$

Note that this angle represents half the swath width. The total swath width is

$$Swath = 2R_e\alpha = R_e\left[2\sin^{-1}\left\{\frac{R_e + h}{R_e}\sin(FOV/2)\right\} - FOV\right]$$

From this table it is clear that for the lowest altitude we will need four satellites, while for the higher two, we will need three satellites.