EE/Ae 157b
Exam
Due: In Class, February 24, 2020

Instructions:

1. **Open book, take home**
2. **No time limit**
3. **No collaboration**

Problem 1. (20 points)

A dihedral corner reflector oriented at an angle α relative to the horizontal axis is characterized by the following scattering matrix

$$[\mathbf{S}] = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ -\sin 2\alpha & -\cos 2\alpha \end{bmatrix}$$

Calculate the covariance matrix for the dihedral with orientation as given above in the backscatter case. Remember that in the *backscatter* case *only*, one can show that for linear media $S_{hv} = S_{vh}$. In this case, we define the scattering vector as

$$\vec{S} = \begin{pmatrix} S_{hh} \\ \sqrt{2}S_{hv} \\ S_{vv} \end{pmatrix}$$

Use this form when doing this problem. We define the covariance matrix of a scatterer as

$$C = \vec{S}\vec{S}^{\dagger}$$

Here the † sign means complex conjugate and transposed.

Now let us consider a collection of dihedral that are oriented statistically in a plane orthogonal to the direction in which the wave is propagation. Assume the dihedrals are uniformly randomly oriented in the interval $[0,2\pi]$.

Calculate the eigenvalues of the covariance matrix.

The covariance matrix for similarly randomly oriented cylinders is (see your second homework)

$$\langle \boldsymbol{C} \rangle = \frac{1}{8}\begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Calculate the eigenvalues and eigenvectors for this case.

Problem 2. (20 points)

A very simple model for the scattering from vegetated areas is constructed considering the vegetation to be a layer of uniformly randomly oriented thin cylinders covering a slightly rough surface. The parameters of the surface are: r.m.s. height = 1 cm, correlation length = 10 cm, correlation function = exponential, incidence angle = 45 degrees, dielectric constant = 20. The vegetation is modeled as uniformly randomly oriented thin wires. The scattering matrix elements for a vertically oriented thin wire are:

$$[\mathbf{S}] = \frac{k_0^2 l^3}{6\left[\ln(4l/a)-1\right]}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

In this equation, l is the cylinder length, and a is the cylinder diameter, and $k_0 = 2\pi/\lambda$.

1. Use the small perturbation model to calculate the covariance matrix of the surface. Note that the S_{hh} and S_{vv} terms are simply the square roots of the radar cross-sections as given by the small perturbation model. Also $S_{hv} = 0$. Calculate the 3x3 covariance matrix. Assume the wavelength is 24 cm.
2. Calculate an expression for the covariance matrix of a uniformly randomly oriented cloud of thin wires for the backscatter direction, i.e. the 3x3 covariance matrix.
3. Next, calculate the total covariance matrix as the sum of the covariance matrices of the ground and the cloud of thin cylinders. Derive the expressions for the radar vegetation index, which is defined as

$$RVI = \frac{8 S_{hv} S_{hv}^*}{S_{hh} S_{hh}^* + S_{vv} S_{vv}^* + 2 S_{hv} S_{hv}^*}$$

4. Plot the RVI as a function of the cylinder length for values of the cylinder length between 10 cm and 20 cm, assuming the cylinder radius is 1 mm. Do you think we can use the RVI as an indication of volume scattering from the vegetation layer?
5. Finally, calculate and plot the eigenvalues of the combined covariance matrix as a function of the cylinder length. Can one tell from the mix of eigenvalues how strong (relatively speaking) the vegetation scattering is?