EE/Ae 157b

Homework #3

Due Date: March 9,2020

Place in Class Mailbox in Moore

Problem 1. (15 points)

The average backscatter covariance matrix of a vegetated area with reflections symmetry can be written as

$$[\mathbf{T}] = \xi \begin{pmatrix} 1 & 0 & \rho \\ 0 & \eta & 0 \\ \rho^* & 0 & \zeta \end{pmatrix}$$

where $\xi = \langle S_{hh} S_{hh}^* \rangle$, $\rho = \langle S_{hh} S_{vv}^* \rangle / \langle S_{hh} S_{hh}^* \rangle$, $\eta = 2 \langle S_{hv} S_{hv}^* \rangle / \langle S_{hh} S_{hh}^* \rangle$, and $\zeta = \langle S_{vv} S_{vv}^* \rangle / \langle S_{hh} S_{hh}^* \rangle$. For randomly oriented thin cylinders, these parameters are found to be $\rho = 1/3$, $\eta = 2/3$ and $\zeta = 1$. Calculate the eigenvalues and entropy for the case of the randomly oriented thin cylinders.

The parameters for a forested area in Shasta Trinity National Forest in northern California measured with the NASA/JPL AIRSAR system are

Parameter	P-band	L-band	C-band
η	0.3301	0.3485	0.2416
ζ	0.6529	0.7122	0.4685
ρ	0.2803+i0.0167	0.3950+i0.0582	0.3669+i0.0016

(a) Calculate the three eigenvalues and the entropy for each frequency.

(b) Next, use the model of a uniformly oriented cloud of cylinders to calculate the fraction of the scattering that can be attributed to scattering by the branches.
(c) Is it reasonable to assume that the scattering is mostly from the branches in the canopy?
(d) Are the branches thin compared to the radar wavelength in each case? Explain your answers.

The wavelength for C-band is 5.6 cm, for L-band is 24 cm, and for P-band is 68 cm.

Problem 2 (20 points)

You have been retained by a mapping company to do a quick assessment of the suitability of radar interferometry for floodplain mapping. The required height accuracy is 30 cm at a horizontal spacing of 1 meter. They own their own corporate jet, which has a wingspan of 20 m that could be used for the baseline of the interferometer. This would create a nominal baseline length of 20 meters and a baseline tilt angle of zero degrees. They also believe that they can buy an X-Band radar (3.2 cm wavelength) at a reasonable price, with the option of buying a Ka-Band radar (0.8 cm wavelength) at a considerably higher price. Finally, they plan to set the data recording window such that they will acquire data between 25 and 60 degrees angle of incidence.

(a) They would like to put GPS antennas at each wingtip to determine the position of each of the two antennas from which they can reconstruct the baseline length and tilt angle. According to their GPS expert, the position of each GPS antenna can be reconstructed to an accuracy of 1 cm. What would be the contribution to the height error at either frequency if they fly the jet at 5 km and 10 km altitudes above the terrain? Calculate the worst case where the error is such that the baseline length change is along the axis of the baseline. In the case of the baseline tilt angle, the worst case would be where the measurement error is in the plane orthogonal to the baseline axis, *i.e.* one antenna is measured higher and the other is measured lower.
(b) What should the signal-to-noise ratio be for each radar if the phase noise contribution to the height error should be less than 10 cm? See the notes on the website for the expression of the error due to phase noise.
(c) Would you recommend radar interferometry to them as a viable solution to their problem? If so, which radar should they buy?

Problem 3 (20 points)

A C-band radar (5.66cm wavelength) is flying at an altitude of 600 km and image a surface at a look angle of 35 degrees. Assume the radius of the earth to be 6380 km. We plan to use this system to study surface subsidence resulting from the extraction of groundwater from wells. Calculate:

1. The range to the earth surface assuming the earth to be spherical.
2. The expected phase shift for a vertical subsidence of 10 cm. Vertical here refers to a change along the local normal to the spherical Earth surface.
3. What is the index of refraction change due to atmospheric moisture that would give an equivalent phase shift?
4. Assume the subsidence as a function of cross-track distance *y* is given by

$$z = 10.exp\left\{-\frac{(y-y_0)^2}{10}\right\}; \quad -10\ km \ \leq y - y_0 \ \leq 10\ km$$

Here *z* is the subsidence measured in the vertical direction, and is measured in centimeters. Plot the expected differential phase as a function of *y*. Note that *y* is measured along the surface of the sphere. Take y_0 to be that cross-track distance at which the look angle is 35 degrees.